V.P.& R.P.T.P.Science College,Vallabh Vidyanagar.
B.Sc.(Semester - V) Internal Test
US05CMTH05 (Number Theory)
Date : 9/10/2019 ; Wednesday 11.00 a.m. to 12.15 p.m. Maximum Marks: 25

Que.1 Fill in the blanks. 5

(1) $(525, 231) =$ .....
(a) 10 (b) 31 (c) 21 (d) 7

(2) ..... is Fermat's number .
(a) 100 (b) 116 (c) 327 (d) 257

(3) 765432 is not divisible by .....
(a) 7 (b) 3 (c) 4 (d) 9

(4) $\phi(m) \leq$ ..... , $\forall \ m > 1$.
(a) m-1 (b) m (c) m+1 (d) m - 2

(5) $2x + 7y \equiv 5(mod\ 12)$ has only ..... solutions.
(a) 1 (b) 2 (c) 12 (d) 5

Que.2 (a) Let g be a positive integer greater than 1 then prove that every positive integer a can can be written uniquely in the form $a = c_n g^n + c_{n-1} g^{n-1} + ... + c_1 g + c_0$, where $n \geq 0$, $c_i \in \mathbb{Z}$, $0 \leq c_i < g$, $c_n \neq 0$. 5

OR

Que.2 (b) If P_n is n^{th} prime number then prove that $P_n < 2^{2^n}$, $\forall\, n \in \mathbb{N}$. 5

Que.3 (a) Prove that every prime factor of F_n $(n > 2)$ is of the form $2^{n+2}t + 1$, for some integer t. 5

OR

Que.3 (b) Prove that $S(a) < a\sqrt{a}$. $\forall\ a > 2$. 5

Que.4 (a) Prove that a general integer solution of $x^2 + y^2 + z^2 = w^2$, $(x, y, z, w) = 1$ is given by $x = a^2 - b^2 + c^2 - d^2$, $y = 2ab - 2cd$. $z = 2ad + 2bc$, $w = a^2 + b^2 + c^2 + d^2$. 5

OR

Que.4 (b) Prove that the equation $x^4 + y^4 = z^2$ has no solution with nonzero positive integers x , y , z . Hence prove that $x^4 - 4y^4 = z^2$ has no nonzero positive integer solution. 5

Que.5 (a) Prove that m is prime iff $\phi(m) + S(m) = mT(m)$. 5

OR

Que.5 (b) Prove that Euler's function is multiplicative function and hence find $\phi(1708)$. 5

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