V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2019-20

Subject : Mathematics | US05CMTH03 | Max. Marks : 25

Metric Spaces

Date: 05/10/2019 | Timing: 11.00 am - 12.15 pm

Q: 1. Answer the following by choosing correct answers from given choices. 5

[1] Every Cauchy sequence is
[A] convergent [B] is not always convergent [C] divergent [D] none

[2] In a metric space (M, ρ), its subsets M and ϕ are
[A] open but not closed [B] closed but not open
[C] open as well as closed [D] neither open nor closed

[3] If a subset A of a metric space M is totally bounded then it is
[A] complete [B] unbounded [C] bounded [D] connected

[4] With absolute value metric $\mathbb{R}$ is
[A] compact [B] complete [C] bounded [D] totally bounded

[5] Every finite subset of a metric space is
[A] unbounded [B] compact [C] dense [D] none

Q: 2. Let (M, d) be a metric space and let $d_1(x, y) = \dfrac{d(x, y)}{1 + d(x, y)}$. Then prove that d_1 is a metric on M 5

OR

Q: 2. If ρ and σ are metrics for M and if there exists $k > 1$ such that

$$\frac{1}{k}\sigma(x, y) \leqslant \rho(x, y) \leqslant k\sigma(x, y), \quad \forall x, y \in M$$

then prove that ρ and σ are eqivalent. 5

Q: 3. Let (M_1, ρ_1) and (M_2, ρ_2) be two metric spaces and let $f : M_1 \to M_2$. Then f is continuous on M_1 if and only if $f^{-1}(G)$ is open in M_1 whenever G is open in M_2. 5

OR

Q: 3. If E is any subset of the metric space M. Then show that $\overline{E}$ is closed. 5

Q: 4. Prove that subset A of $\mathbb{R}$ is totally bounded *iff* A is bounded. 5

OR

Q: 4. State and prove generalized nested interval theorem. 5

Q: 5. If A is a closed subset of the compact metric space (M, ρ), then prove that the metric space (A, ρ) is also compact. 5

OR

Q: 5. If the metric space M has the Heine-Borel property, then prove that M is compact. 5