V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2019-20

Subject : Mathematics   US05CMTH02   Max. Marks : 25

Real Analysis-II

Date: 03/10/2019   Timing: 11.00 am - 12.15 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices.   **5**

[1] The sequence $\{S_n\}_{n=1}^{\infty}$, where $S_n = (-1)^n \left(1 + \frac{1}{n}\right)$
[A] is convergent [B] oscillates finitely [C] oscillates infinitely [D] is divergent

[2] A positive term series $\sum\limits_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if and only if
[A] $p < 1$ [B] $p > 1$ [C] $p \leqslant 1$ [D] $p \geqslant 1$

[3] For $f(x,y) = x^3 - 3xe^y$ the value of $f_x(1,0)$ is
[A] 0   [B] 1   [C] 2   [D] 3

[4] For a function f, if $f_x(a,b) < f_y(a,b)$ then at (a,b), f has
[A] no extreme value [B] a minimum [C] a maximum [D] a stationery point

[5] For a function f, if $f_x(1,5) = 0$, $f_y(1,5) \neq 0$ then at $(1,5)$, f has
[A] an extreme value [B] no extreme value [C] a minimum [D] a maximum

Q: 2. State and prove the Bolzano-Weierstarss theorem for sequence   **5**

OR

Q: 2. Show that the sequence $\{r^n\}$ converges iff $-1 < r \leqslant 1$.   **5**

Q: 3. State and prove the comparision test of first type.   **5**

OR

Q: 3. State and prove D' Alembert's ratio test.   **5**

Q: 4. Show that $\lim\limits_{(x,y)\to(0,0)} xy\frac{x^2 - y^2}{x^2 + y^2} = 0$ **5**

OR

Q: 4. Prove that, by the transformation $u = x - ct$, $v = x + ct$, the partial differential equation $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$ reduaces to $\frac{\partial^2 z}{\partial u \partial v} = 0$ **5**

Q: 5. State and prove Taylor's theorem   **5**

OR

Q: 5. Find the maxima and minima of the function $x^3 + y^3 - 3x - 12y + 20$ **5**

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