V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2019-20

Subject : Mathematics US05CMTH01 Max. Marks : 25

Real Analysis-I

Date: 01/10/2019 Timing: 11.00 am - 12.15 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. **5**

[1] The infimum of the set $-1, 1, -1\frac{1}{2}, 1\frac{1}{2}, -1\frac{1}{3}, 1\frac{1}{3}, \ldots$

[A] -1 [B] 0 [C] $-1\frac{1}{2}$ [D] $\frac{1}{2}$

[2] If $S = (1,5) - \{3\}$, then 3 is

[A] a limit point of S

[B] an interior point of S

[C] interior point as well as limit point of S

[D] none

[3] If a function $f(x)$ has a discontinuity of first type at $x = 2$ then $\lim\limits_{x\to 2-} f(x)$ and $\lim\limits_{x\to 2+} f(x)$ both

[A] do not exist [B] exist and they are equal

[C] exist but they are not equal [D] cannot exist togather

[4] If $f'(1) = 5$ then at $x = 1$ function f is

[A] increasing [B] decreasing [C] discontinuous [D] not derivable

[5] If f is continuous on an interval I then

[A] f is uniformly continuous on I

[B] f is not necessarily uniformly continuous on I

[C] f may have some points of discontinuities in I

[D] none

Q: 2. State the Least Upper Bound property of R and prove that the field of rational numbers is not order complete. **5**

OR

Q: 2. Define Exponential Function. Also state and prove the addition formulae for exponential function. **5**

Q: 3. State and prove the *Bolzano-Weiestrass* theorem for sets. **5**

OR

Q: 3. Prove that derived set of a set is closed. **5**

Q: 4. Let f and g be two functions defined on some neighbourhood of a such that $\lim_{x \to a} f(x) = l$ and $\lim_{x \to a} g(x) = m$. Prove that $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{l}{m}$, if $m \neq 0$ **5**

OR

Q: 4. Show that a function $f : [a, b] \to \Re$ is continuous at point c of [a,b] iff

$$\lim_{n \to \infty} c_n = c \Longrightarrow \lim_{n \to \infty} f(c_n) = f(c)$$

5

Q: 5. If $f'(c) < 0$, then prove that f is a monotonic decreasing function at point $x = c$. **5**

OR

Q: 5. Show that $\log(1 + x)$ lies between $x - \frac{x^2}{2}$ and $x - \frac{x^2}{2(1+x)}$, $\forall x > 0$ **5**