V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2018-19

Subject : Mathematics   US06CMTH05   Max. Marks : 50

Graph Theory

Date: 11/03/2019   Timing: 10:00 am - 12:00 Noon

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. **8**

[1] The maximum number of edges in a simple graph with 5 vertices and 2 components is
[A] 3 [B] 4 [C] 5 [D] 6

[2] If terminal vertices in a walk are not same then it is called
[A] an open walk [B] closed walk [C] null graph [D] none

[3] A minimally connected graph is
[A] a tree [B] a circuit [C] an Euler graph [D] none

[4] An operation of vertex deletion on a graph removes corresponding
[A] edges only [B] vertices only [C] vertex and edges both [D] none

[5] A spanning tree in a graph G must contain all the
[A] vertices of G [B] edges of G [C] circuits of G [D] paths of G

[6] If rank of a matrix is 9 and its nullity is 5 then the number of its edges is ___
[A] 4 [B] 5 [C] 9 [D] 14

[7] The number of faces in a simple connected planar graph with 8 edges and 6 vertices is
[A] 2 [B] 4 [C] 6 [D] 8

[8] If G_1 and G_2 are isomorphic and rank of G_2 is 5 then the rank of G_1 is
[A] 25 [B] 5 [C] 10 [D] *none*

Q: 2. Answer any FIVE of the following. **10**

[1] What is graph? Explain it with example.

[2] Define (i) Path (ii) Closed walk

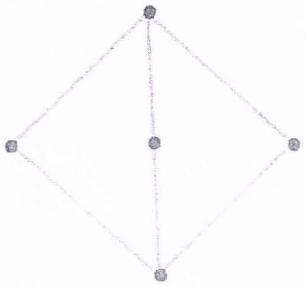
[3] Explain Union of two graphs with an example.

[4] Explain Arbitrarily Traceable Graphs with an example.

[5] Prove that the edge connectivity of a graph G can not exceed the degree of a vertex with the smallest degree in G.

[6] Describe network flows

[7] Find geometric dual of the following graph



G:

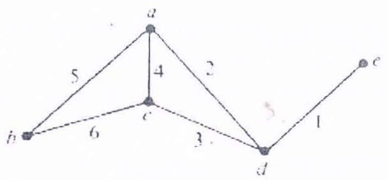
[8] Discuss Kurtowski's First graph.

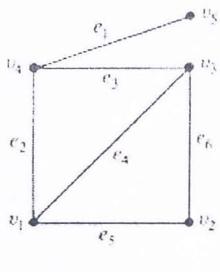
Q: 3 [A] Prove that a simple graph with n vertices and k-components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. 5

[B] If a graph (connected or disconnected) has exactly two vertices of odd degree then prove that there must be a path joining these two vertices. 3

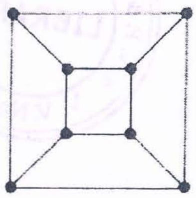
OR

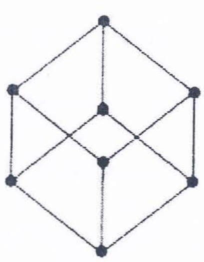
Q: 3 [A] Explain Isomorphism between two graphs.. Also examine whether following pairs of graphs are isomorphic or not. 5





[1]





[2]

[B] Discuss Konigsberg bridge problem 3

Q: 4 [A] Prove that a connected graph G is an Euler graph *iff* all vertices of G are of even degree. 5

[B] Prove that any connected graph with n-vertices and $n-1$ edges is a tree. 3

OR

Q: 4 [A] Prove that every connected graph G is an Euler graph *iff* it can be decomposed into circuits. 5

[B] If in a graph G there is one and only one path between every pair of vertices then prove that G is a tree. 3

Q: 5 [A] Prove that every connected graph has atleast one spanning tree. 5

[B] Describe a method to find all spanning tree of a graph. 3

OR

Q: 5 [A] Prove that every cut-set in a connected graph G must contain atleast one branch of every spanning tree. 5

[B] Prove that every circuit has an even number of edges in common with any cut-set. 3

Q: 6 [A] Using geometric arguments prove that $K_{3,3}$is non-planar. 5

[B] For a simple connected planar graph with n-vertices , e-edges ($e > 2$) and f-regions prove the following.
(i) $e \geqslant \frac{3}{2}f$ (ii) $e \leqslant 3n - 6$ 3

OR

Q: 6 [A] Give an example to show that two isomorphic graphs may not have isomorphic duals. 5

[B] Explain Geometric dual with an example 3