V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
B.Sc. (Semester - VI) Internal Test
US06CMTH04 (Abstract Algebra - 2)

Date. 9/3/2019 ; Saturday 10.00 a.m. to 12.00 noon Maximum Marks: 50

Que.1 Fill in the blanks. 8

(1) .... is regular element of $\mathbb{Z}_{20}$.

(a) 10 (b) 4 (c) 6 (d) none of these

(2) Let R be the set of all subsets of a set X . Define $+ \; and \cdot$ in R by $A + B = (A - B) \cup (B - A)$, $A \cdot B = A \cap B$, then Ch R ....

(a) 1 (b) 2 (c) 0 (d) ϕ

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(3) Every integral domain can be imbedded in a ....

(a) $\mathbb{Z}$ (b) $\mathbb{N}$ (c) field (d) ring

(4) .... is an ideal in ring R .

(a) 0 (b) $\{1\}$ (c) $\{0\}$ (d) none of these

(5) In $R = \{a + b\sqrt{-5}/a, b \in \mathbb{Z}\}$, $1 + 2\sqrt{-5}$ is .... in R .

(a) unit (b) irreducible (c) prime (d) non of these

(6) Cancelation laws are always satisfied in ....

(a) integral domain (b) ring (c) ring with unit element (d) commutative ring

(7) If F is field then $F[x]$ is ....

(a) $\{0\}$ (b) F (c) principle ideal domain (d) field

(8) If F is field, $f(x) \in F[x]$, degree of f is n then $f(x)$ has ....distinct roots in F .

(a) 2 (b) atleast n (c) n (d) atmost n

Que.2 Answer the following (Any Five) 10

(1) Prove that ring $C[0, 1]$ is not an integral domain.

(2) Prove that the mapping $f : C \to C$ defined by $f(z) = \bar{z}$ is a ring isomorphism.

(3) Prove that $\{ \bar{0}, \bar{2}, \bar{4} \}$ is an ideal in Z_6 .

(4) Prove that an ideal pZ is a maximal ideal in Z , where p is a prime number.

(5) Let$R = \{a + b\sqrt{-5}/a, b \in Z\}$.Show that $1 + 2\sqrt{-5}$ and 3 are relatively prime.

(6) If R is ring with unit element 1 then prove that $Ra \subset Rb \Leftrightarrow b/a$.

(7) Prove that every principal ideal domain is a unique factorization domain.

(8) Find all roots of $x^3 + 5x$ in Z_6.

Que.3 (a) Prove that $R = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} / \ a, b, \in \mathbb{R} \right\}$ is a commutative ring with unit element.Also prove t
it is field and integral domain .

OR

Que.3 (b) State and prove Cayley's theorem for rings.

(c) P.ove that the characteristic of a field is either 0 or a prime. 4

Que.4 (a) Prove that every maximal ideal in a commutative ring with 1 , is a prime ideal . Under which condition converse hold ? Verify it . 4

(b) State and prove First isomorphism theorem for ring. 4

OR

Que.4 (c) Prove that P is prime ideal of $\mathbb{Z}$ iff either P = 0 or $P = p\mathbb{Z}$, for some prime number p . 4

(d) Give an example of ring which is simple ring but not a field. Verify it. 4

Que.5 (a) Prove that every prime element is irreducible in integral domain with unit element 1 . Does the converse hold ? Verify it. 8

OR

Que.5 (b) Prove that every principal ideal domain is factorization domain. 5

(c) Let$R = \{a + b\sqrt{-5}/a, b \in Z\}$.Find gcd of $\alpha\gamma$ and$\beta\gamma$ in R , where $\alpha = 3$; $\beta = 1 + 2\sqrt{-5}$; $\gamma = 7(1 + 2\sqrt{-5})$. 3

Que.6 (a) State and prove Gauss lemma and Gauss theorem . 8

OR

Que.6 (b) If F is a field then prove that F[x] is a Euclidean domain. 5

(c) Let R be an integral domain in which : 3
(i) every $a \in R$ which is non-unit can be expressed as a product of irreducible elements.
(ii) Every irreducible element is prime.
Then prove that R is unique factorization domain.

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