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Internal Test: 2018-19

Subject : Mathematics US06CMTH03 Max. Marks : 50

Topology

Date: 08/03/2019 Timing: 10:00 am - 12:00 Noon

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. **8**

[1] In a topological space $(X, \mathcal{T})$, a neighbourhood of a point is
[A] $\mathcal{T}$-open [B] $\mathcal{T}$-closed [C] either open or closed [D] none

[2] Any topology on a non-empty set is ------- the indiscrete topology on that set.
[A] coarser than [B] finer than [C] non-comparable [D] none

[3] In $(\mathcal{R}, \mathcal{U})$ which of the following is not closed
[A] $\emptyset$ [B] $\mathcal{R}$ [C] $(1, 2)$ [D] $[1, 2]$

[4] If A is a dense subset of a topological space $(X, \mathcal{T})$ then
[A] $A' = X$ [B] $A = X$ [C] $A^- = X$ [D] none

[5] If there is a proper subset of a topological space $(X, \mathcal{T})$ which is open as well as closed then $(X, \mathcal{T})$ is a
[A] compact space [B] connected space
[C] disconnected space [D] none

[6] In its relativised topology, the subset ---- of R is disconnected.
[A] $(0, 1)$ [B] $[0, 1]$ [C] $(0, 1) \cup (1, 2)$ [D] $(0, 1) \cup [1, 2)$

[7] If every open cover of a topological space has a finite subcover then it is
[A] Compact [B] Unbounded [C] a Regular Space [D] none

[8] In a T_1 space the complement of every singleton set is
[A] closed [B] open [C] closed and open both [D] neither open not closed

Q: 2. Answer any FIVE of the following. **10**

[1] Show that the sets $\mathbb{R}$ and $\emptyset$ are $\mathcal{U}$-open

[2] Define (i) Topological Space (ii) Usual Topology of $\mathbb{R}$

[3] Find the sets of cluster points of $(1, 2)$ in usual topology and discrete topology of $\mathbb{R}$

[4] Define (i) Closure of a set (ii) Interior Point

[5] Prove that indiscrete space is connected

[6] For $X = \{a, b, c\}$ consider the topology $\mathcal{T} = \{X, \emptyset, \{a, b\}, \{c\}\}$. Is $(X, \mathcal{T})$ connected?

[7] Is every discrete space a T_1 space also? Why?

[8] Prove that every metric space is a Hausdorff space

Q: 3 [A] Show that usual topology of $\mathbb{R}$ possesses all the properties for becoming a topology for $\mathbb{R}$ **5**

[B] Consider the topology $\mathcal{G}$ on $\mathbb{R}$ where $G \subset \mathbb{R}$ is $\mathcal{G}$-open if $G = \emptyset$ or $G \neq \emptyset$ and for each $p \in G$ there is a set $H = \{x \in \mathbb{R} / a \leqslant x < b\}$ for some $a < b$ such that $p \in H \subset G$. Prove that $\mathcal{G}$ is finer than usual topology of $\mathbb{R}$ **3**

OR

Q: 3 [A] Define Closed Set. Also if $(X, \mathcal{T})$ is a topological space and $F_1, F_2, ..., F_n$ are $\mathcal{T}$-closed subsets of X then prove that $\bigcup\{F_i / i \in J_n\}$ is a $\mathcal{T}$-closed set **5**

[B] Are closed intervals of $\mathbb{R}$, $\mathcal{U}$-closed? where $\mathcal{U}$ is the usual topology for $\mathbb{R}$ **3**

Q: 4 [A] Let $(X, \mathcal{T})$ be a topological space and A be a subset of X. Prove that $A \cup A'$ is $\mathcal{T}$-closed **5**

[B] Find the sets of cluster points of $\mathcal{R}$ and $\{\frac{1}{n} / n \in J^+\}$ subsets of $\mathbb{R}$ relative to $\mathcal{U}$-topology and $\mathcal{I}$-topology **3**

OR

Q: 4 [A] Let $(X, \mathcal{T})$ and (Y, Ψ) be topological spaces and f be a mapping from X into Y. Prove that if $f(A^-) \subset f(A)^-$ for $A \subset X$, then the inverse image of f of every Ψ-closed set is $\mathcal{T}$-closed set. **5**

[B] Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X. Then prove that $A^- = A \cup A'$. **3**

Q: 5 [A] Show that a relative topology satisfies all the conditions for becoming a topological space **5**

[B] Prove that the space $(R, \mathcal{U})$ is not compact and hence prove that no open interval is compact in its relativized $\mathcal{U}$ topology. **3**

OR

Q: 5 [A] Assuming that connectedness is a topological property prove that $(R, \mathcal{U})$ and $(R, \mathcal{G})$ are not homeomorphic where $\mathcal{U}$ is usual topology for R and $\mathcal{G}$ is defined as follows
$G \in \mathcal{G}$ if either G empty or it is a nonempty subset of R such that for every $p \in G$ there is some $H = \{x \in R / a \leqslant x < b\}$ for $a < b$ such that $p \in H \subset G$. **5**

[B] If $(X, \mathcal{T})$ is compact and Y is a $\mathcal{T}$-closed subset of X, then prove that $(Y, \mathcal{T}_Y)$ is also compact. **3**

Q: 6. State and prove the Heine-Borel theorem. **8**

OR

Q: 6 [A] Prove that every compact Hausdorff space is a T_3-space. **5**

[B] If $(X, \mathcal{T})$ is a compact space, and if f is a $\mathcal{T} - \psi$ continuous mapping of X into R, then prove that f is bounded. **3**

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