V.P.& R.P.T.P.Science College,Vallabh Vidyanagar.
B.Sc.(Semester - VI) Internal Test
US06CMTH02 (Complex Analysis)
Date. 6/3/2019 ; Wednesday 10.00 a.m. to 12.00 noon Maximum Marks: 50

Que.1 Fill in the blanks. 8

(1) $f(z) = (x^2 - y^2 - 2y) + i(2x - 2xy)$ can be expressed as $f(z) =$........................

(a) $\bar{z}^2 + 2z$ (b) $\bar{z}^2 + iz$ (c) $\bar{z}^2 - 2iz$ (d) $\bar{z}^2 + 2iz$

(2) Domain of $f(z) = z + \dfrac{1}{z}$ is ....

(a) $\mathbb{C} - \{0\}$ (b) $\mathbb{C} - \{i\}$ (c) $\mathbb{C} - \{\pm 1\}$ (d) $\mathbb{C} - \{\pm 1\}$

(3) If $f(z) = 2x + iy^2x$ then f is differentiable at ....

(a) 2 (b) 1 (c) 0 (d) none of these

(4) $f(z) = \dfrac{z^3 + i}{(z^2 - 3z + 2)}$ is analytic in ....

(a) $\{\pm\sqrt{1} , \pm 2\}$ (b) $\mathbb{C} - \{1 , 2\}$ (c) $\mathbb{C} - \{3 , \pm 2\}$ (d) $\{ 1 , 2 \}$

(5) e^z is periodic function with period .... , $n \in \mathbb{Z}$

(a) $n\pi i$ (b) $2n\pi$ (c) $2n\pi i$ (d) $(2n+1)\pi i$

(6) $cosxcoshy - isinxsinhy =$ ....

(a) $cosz$ (b) $sinz$ (c) $sinhz$ (d) $coshz$

(7) Image of $y > 1$ under the transformation $w = (1 - i)z$ is ....

(a) $u + v < 2$ (b) $v + u > 2$ (c) $u - v > 2$ (d) $u - v < 2$

(8) Fixed point of $w = \dfrac{6z - 9}{z}$ are ....

(a) 0 (b) i (c) 2 (d) 3

Que.2 Answer the following (Any Five) 10

(1) Prove that limit of function is unique,if it exist.

(2) By using definition, prove that $\dfrac{d}{dz}(z^{-1}) = -\dfrac{1}{z^2}$.

(3) Verify that $f(z) = \cosh x \cos y + i \sinh x \sin y$ is entire or not.

(4) Find a harmonic conjugate $v(x, y)$ for harmonic function $u(x, y) = 2x(1 - y)$.

(5) Find all values of z such that $e^z = -1 - \sqrt{3}i$.

(6) Prove that $|\sinh x| \leq |\cosh z| \leq \cosh x$.

(7) Prove that $w = Az$,where A is non-zero complex constant,$z \neq 0$, gives an expansion or contraction by $|A|$ or rotation through argA about origin.

(8) Find the image of $0 < x < 1$, $0 < y < 1$ under the transformation $w = iz$. Also sketch the region .

Que.3 (a) By using definition of limit prove that $\lim_{z \to 2i} (2x + iy^2) = 4i$.

(b) State and prove chain rule for differentiating composite functions.

OR

Que.3 (c) Give an example of function such that its real and imaginary component have continuous partial derivative of all order at a point but the function is not differentiable at that point. Verify it. 5

(d) If $\lim_{z \to z_0} f(z) = w_0$; $\lim_{z \to z_0} g(z) = w_1$. Then prove that $\lim_{z \to z_0} \frac{f(z)}{g(z)} = \frac{w_0}{w_1}$ $if\ w_0 \neq 0$ 3

Que.4 (a) Let $f(z) = u(x, y) + iv(x, y)$ and f'(z) exist at $z_0 = x_0 + iy_0$. Prove that the first order partial derivatives of u and v must exist at (x_0, y_0) and they satisfies the Cauchy-Reimann equations $u_x = v_y$; $u_y = -v_x\ at\ (x_0, y_0)$. Also prove that $f'(z) = u_x + iv_x$ where $u_x\ and\ v_x$ are evaluated at (x_0, y_0). Does the converse of above result holds? Verify it. 8

OR

Que.4 (b) State and prove sufficient conditions for differentiability of $f(z)$. 6

(c) Prove that $f'(z)$ does not exist at any point for $f(z) = 2x + ixy^2$ 2

Que.5 (a) Prove that $(e^z)^n = e^{nz}$ $\forall$ $n \in \mathbb{Z}$. 4

(b) Solve the equation $\cos w = \sqrt{2}$. 4

OR

Que.5 (c) Prove that $cos z_1 - cos z_2 = -2 sin\left(\frac{z_1 + z_2}{2}\right) sin\left(\frac{z_1 - z_2}{2}\right)$. 4

(d) Solve the equation $\cosh z = 1/2$ 4

Que.6 (a) Discuss the image of $w = (i + 1)z + 2$. Hence sketch the rectangle $1 \leq x \leq 2$, $1 \leq y \leq 4$ and its image. 4

(b) Find the image of semi infinite strip $x > 0$, $0 < y < 1$ under the transformation $w = i/z$. Also sketch the strip and its image. 4

OR

Que.6 (c) Find linear fractional transformation that maps the points $z_1 = -i$, $z_2 = 0$, $z_3 = i$ $onto$ $w_1 = -1$, $w_2 = i$, $w_3 = 1$ respectively. 4

(d) Prove that all linear fractional transformation that maps the upper half plane $Im\ z > 0$ onto the open disk $|w| < 1$ and the boundary $Im\ z = 0$ on to the boundary of $|w| = 1$ is given by $w = e^{i\alpha}\left[\frac{z - z_0}{z - \overline{z_0}}\right]$, $(Im\ z_0 > 0)$. 4

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