V.P.& R.P.T.P. Science College ,Vidyanagar.
B.Sc.(SEMESTER - V) Internal Test
MATHEMATICS : US05CMTH06 (Mechanics - 1)
Date. 8/10/2018 ; Monday 10.00 a.m. to 12.00 noon Maximum Marks: 50

Que.1 Fill in the blanks. 8

(1) If $V = x^2 + y^2$ then component of grad V at point (0,1) in the direction making angle $45^0$with X axis is .....
(a) $\sqrt{2}\,\bar{i}$ (b) $-\sqrt{2}\,\bar{i}$ (c) $\sqrt{2}\,\bar{j}$ (d) $-\sqrt{2}\,\bar{j}$

(2) [Angular Momentum] = ...
(a) ML^2T^{-2} (b) ML^2T^{-1} (c) ML^2T (d) ML^2T^{-1}

(3) A branch of mechanics which deals with the equilibrium of systems at rest is known as .....
(a) dynamics (b) statics (c) motion (d) acceleration

(4) If line of action of $\bar{P}$ intersect the line L then moment about L = ...
(a) 0 (b) 2 (c) 1 (d) -1

(5) Dimension of power is .....
(a) MLT^{-2} (b) ML^2T^{-2} (c) ML^2T^2 (d) ML^2T^{-3}

(6) If density ρ is constant then the body is said to be .....
(a) homogeneous (b) heterogeneous (c) exact (d) rigid

(7) Tension for catenary is given by T = ...
(a) $wy - H$ (b) $Hy + w$ (c) $\sqrt{H^2 + w_0^2x^2}$ (d) $wy + H$

(8) Radial component of acceleration of a particle moving in a plane is .....
(a) $\ddot{r} + r\,\dot{\theta}^2$ (b) $\ddot{r} - r\,\dot{\theta}$ (c) $\dot{r} - r\,\dot{\theta}^2$ (d) $\ddot{r} - r\,\dot{\theta}^2$

Que.2 Answer the following (Any Five) 10

(1) Express a mass of 14.3 oz in cgs system .
(2) Two forces acting in opposite direction on a particle have a resultant of 34 lbwt. If they act at right angle to one-another, their resultant would be of 50 lbwt. Find magnitude of the two forces.
(3) ABCD is a square of side 2 unit, forces 1 , 2 , 3 , 4 lb wt act along $\overline{AB}, \overline{CB}, \overline{DC}, \overline{DA}$ respectively . Find the algebraic sum of their moments about Center of a square .
(4) Find the horizontal force required to keep a particle of mass 2 gm at rest on a smooth plane inclined an angle 30^o with horizontal .
(5) In usual notations prove that : $\delta W = X\delta x + Y\delta y + Z\delta z$.
(6) A Forces of magnitude 5 , 1 , 1 , 3 , $4\sqrt{2}$ along the sides AB , BC , CD , DA and diagonal BD of a square $ABCD$ respectively . Taking $AB$and AD as x and y axes respectively .Find the magnitude of resultant force .
(7) In usual notation prove that $s^2 = y^2 + 2cy$.
(8) Prove that tension is constant along a light cable in contact with smooth curves.

Que.3 (a) State and prove Law of Parallelogram of two forces .Hence find resultant force when two forces are acting (i) along the same line and same direction (ii) along the same line and opposite direction (iii) along right angles. 5

(b) A vertical force of 10 kg wt is resolve into two equal component , one of them making an angle of 30^o with vertical ,find magnitude and direction of the other. 3

OR

Que.3 (c) If the fundamental law of mechanics of a particle moving on a straight line is 5

$m\frac{d}{dt}\left(\frac{\dot{x}}{\sqrt{1-\frac{\dot{x}^2}{c^2}}}\right) = F$. Find the distance traveled from the rest in time 't' under the action of a force F .

(d) Find the component of gradient of V along a co-ordinate axis . 3

Que.4 (a) State and prove theorem of Varignor 4

(b) A door of weight w , height 2a , width 2b is hanged at top and bottom .If the reaction at upper hinge has no vertical component , find the components of reaction at both hinge ,assume that the weight of the door acts at it's center . 4

OR

Que.4 (c) A particle of weight w is suspended from a fixed point by a light string .A horizontal force H is applied to it and the particle takes up a position of equilibrium with the string inclined to a vertical .If the string breaks when the tension in it reaches at value T_0 ,find the smallest value of H necessary to break the string . 5

(d) Three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ acting at a point are in equilibrium and the angle between $\vec{P}$ and $\vec{Q}$ is doubled of angle between $\vec{P}$ and $\vec{R}$. Prove that $R^2 = Q(Q - P)$. 3

Que.5 (a) Prove that the force of attraction of a thin spherical shell at any 8
(i) external point of shell is directed toward the centre and magnitude of force is GM/r^2 .
(ii) internal point of shell is zero .

OR

Que.5 (b) A rod AB is movable about point A , and at B attached a string whose other end is tied to a ring. The ring slides on a smooth horizontal wire passing through A . By using principle of virtual work prove that horizontal force necessary to keep the ring at rest is $\frac{w \cos\alpha \cos\beta}{2 \sin(\alpha+\beta)}$, where w is weight of the rod , α and β are the inclination of the rod and the string to the horizontal. 4

(c) If two heavy particles of weight w , w' are connected by a light inextensible string and hang over a fixed smooth circular cylinder of radius a , the axis of which is horizontal . If system is in equilibrium then prove that $\frac{\sin\theta}{\sin\theta'} = \frac{w'}{w}$. 4

Que.6 (a) Derive the general formula for the flexible cable hanging freely . 3

(b) Derive differential equation of suspension bridge. Also show that it represent the equation of parabola and find its tension . 5

OR

Que.6 (c) A particle moves in a catenary $S = c\tan\psi$. The direction of its acceleration at a point makes equal angle with the tangent and normal to the path at the point. If the speed at the vertex where $\psi = 0$ is u then show that the velocity and resultant acceleration at any point are given by ue^{ψ} and $\frac{\sqrt{2}u^2e^{2\psi}\cos^2\psi}{c}$ respectively. 4

(d) A uniform cable is hanging across two smooth pegs at the same height. The ends are hanging down vertically. If the free ends are each 12 ft. long and the tangents to the catenary at each pegs makes an angle 60^o with the horizontal line. Find the total length of the cable. 4

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