V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2018-19

Subject : Mathematics | US05CMTH01 | Max. Marks : 50

Real Analysis-I

Date: 29/09/2018 | Timing: 10.00 am - 12.00 pm

Q: 1. Answer the following by choosing correct answers from given choices. **8**

[1] The smallest member of a set S, if exists, is
[A] the supremum of S [B] the infimum of S [C] not unique [D] none

[2] The infimum of the set $-1, 1, -1\frac{1}{2}, 1\frac{1}{2}, -1\frac{1}{3}, 1\frac{1}{3}, \dots$
[A] -1 [B] 0 [C] $-1\frac{1}{2}$ [D] $\frac{1}{2}$

[3] The interior of the set of integers is
[A] N [B] Q [C] R [D] ϕ

[4] If S_1 and S_2 are closed sets then $S_1 \cup S_2$ is
[A] closed [B] open [C] Open as well as closed [D] none

[5] If $\lim_{x\to a} f(x)$ exists but $f(a)$ does not exist then f posseses a discontinuity of
[A] removable type [B] first type [C] second type [D] first type from left

[6] $\lim_{x\to 2-} [x] - x =$
[A] 0 [B] -1 [C] -2 [D] -3

[7] If f is uniformly continuous on an interval then on that interval it is
[A] continuous [B] discontinuous [C] differentiable [D] not differentiable

[8] If $f'(1) = 5$ then at $x = 1$ function f is
[A] increasing [B] decreasing [C] discontinuous [D] not derivable

Q: 2. Answer Any FIVE of the following. **10**

[1] Find the Greatest and the smallest members of $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots\right\}$ if they exist.

[2] Find the l.u.b and smallest member of $\{n^2 \ / n \in N\}$ if they exist.

[3] Determine whether the interior of the set $[2, 8] \cup (9, 10) \cap N$ is open or not.

[4] Give an example of a set which is neither open nor closed.

[5] If $[x]$ denotes the largest integer less than or equal to x, then discuss the continuity at $x = 3$ for the function $f(x) = x - [x]$, $\forall x \geqslant 0$,

[6] Evaluate $\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x}$

[7] Prove that the function x^2 is derivable on $[0, 1]$.

[8] If f is derivabale at c and $f(c) \neq 0$ then the function $\frac{1}{f}$ is also derivable thereat, and $\left(\frac{1}{f}\right)' = -\frac{f'(c)}{\{f(c)\}^2}$

Q: 3 [A] State the Least Upper Bound property of R and prove that the field of rational numbers is not order complete. 5

[B] State and prove the addition formulae for exponential function. 3

OR

Q: 3 [A] Define generalized power function and in usual notations prove that:

$$(i)\ a^{x+y} = a^x.a^y \qquad (ii)\ a^n = \underbrace{a.a.....a}_{n\ times} \qquad (iii)\ a^{-n} = \underbrace{\frac{1}{a}.\frac{1}{a}.\frac{1}{a}.....\frac{1}{a}}_{n\ times}$$

5

[B] Prove that $\sqrt{11}$ is not a rational number. 3

Q: 4 [A] Prove that interior of a set S is the largest open subset of S. 5

[B] If S and T are sets of real numbers then prove the following
(i) $S \subset T \Rightarrow S' \subset T'$ (ii) $(S \cup T)' = S' \cup T'$ 3

OR

Q: 4 [A] Prove that a set is closed *iff* its complement is open. 5

[B] Prove that derived set of a bounded set is bounded. 3

Q: 5 [A] Let f and g be two functions defined on some neighbourhood of a such that $\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$. Prove that $\lim_{x\to a} [f(x)g(x)] = lm$ 5

[B] If a function is continuous on a closed interval $[a, b]$, then it attains its bounds at least once in $[a, b]$. 3

OR

Q: 5 [A] If a function f is continuous at an interior point c of $[a, b]$ and $f(c) \neq 0$, then prove that, there exists $\delta > 0$ such that $f(x)$ has the same sign as $f(c)$ for every $x \in (c-\delta, c+\delta)$. 5

[B] Evaluate $\lim_{x\to 0} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1}$ 3

Q: 6 [A] If $f'(c) > 0$, then prove that f is a monotonic increasing function at point $x = c$. 5

[B] Prove that the function $\frac{1}{x}$ is not uniformly continuous on $(0, 1]$. 3

OR

Q: 6 [A] Prove that a function which is derivable at a point is necessarily continuous at that point. Is the converse true? Justify. 5

[B] Prove that, $\frac{x}{1+x} < \log(1+x) < x$ for all $x > 0$. 3