V.P. & R.P.T.P. Science College, Vallabh Vidyanagar

Date : 3/10/2019 | B.Sc. Semester - III

Time: 3.00 to 4.15 pm | Internal Test – 2019 | M.Marks : 25

Subject : Statistics | Course Code : US03CSTA22 | Elements of Probability Theory

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$Q.1$ Multiple Choice Questions

(1) If $P(A) = 0.7, P(B) = 0.2, P(A' \cup B') = 0.9$, then prob. that at least one of the event occurs is

(a) 0.9 (b) 0.5 (c) 0.2 (d) 0.8

(2) The joint pdf of X and Y is $f(x,y) = \frac{xy}{4}, 0 < x < 2, 0 < y < 2$ *and zero otherwise*, then $P(X < 1)$ *is*

(a) 1/4 (b) 3/8 (c) 1 (d) 1/8

(3) The pdf of a r.v. X is then $P(X > 1/3) =$ _____

$$f(x) = \begin{cases} 1 - |x|, -1 < x < 1 \\ 0, otherwise \end{cases}$$

(a) 2/9 (b) 11/18 (c) 13/18 (d) None of these

(4) If $M_x(t) = e^{2(e^t - 1)}$ is the m.g.f of a random variable X then $V(3 + 2X) =$ = _____

(a) 7 (b) 2 (c) 8 (d) 4

(5) Let X and Y have the joint prob. mass function

$f(x,y) = \frac{x+y}{21}, x = 1,2,3 ; y = 1,2$ *and zero otherwise*, then $P(X = 2/Y = 2)$ equals

(a) 1/3 (b) 2/3 (c) 1/2 (d) 1/4

$Q.2$ For two events A and B, Prove that

$(i)\ P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$ $(ii)\ P(A \cap B) \geq P(A) + P(B) - 1$

OR

$Q.2$ Let A and B be two independent events. The prob. of simultaneous occurrence of these events is $1/8$ and the probability of none of these occurs is $3/8$. Find $P(A)$ and $P(B)$.

$Q.3$ If $f(x) = \begin{cases} kx, 0 \leq x < 3 \\ k(6-x), 3 \leq x < 6 \\ 0, x \geq 6 \end{cases}$

is the pdf of X then find $(i)\ k$ (ii) the c.d.f of X $(iii)\ P\left(\frac{1}{2} < X < 4\right)$

OR

$Q.3$ An urn contains 5 white and 2 black marbles. If 3 marbles are to be selected at random and let X denote the no. of black marbles, find the probability distribution and c.d.f of X. Determine the prob. that a randomly selected marbles have atleast one black marble.

$Q.4$ The probability distribution of X is

If $f(x) = \frac{1}{10}, -5 < x < 5$ *and zero otherwise*. Find the m.g.f of X and hence mean and variance of X.

OR

$Q.4$ A continuous random variable X in the range $(-3, 3)$ is given by the pdf

$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2, -3 \leq x < -1 \\ \frac{1}{16}(6 - 2x^2), -1 \leq x \leq 1 \\ \frac{1}{16}(3-x)^2, 1 \leq x \leq 3 \end{cases}$$

(i) Verify that the area under the curve is unity (ii) Find $(a) E(2X)$ $(b)\ P(1/2 < X < 2)$

$Q.5$ Let X and Y have the joint density function

$f(x,y) = x + y, 0 < (x,y) < 1$ *and zero otherwise*

(i) Find the correlation coefficient between X and Y (ii) Are X and Y independent?

(iii) Calculate $P(X > 1/3)$

OR

$Q.5$ The joint probability distribution of X and Y is

$f(x,y) = K(x+y)$, $x = 1,2$ *and* $y = 1,2,3,4$ *and zero otherwise.*

Find (i) the value of K $(ii)\ V(X - Y)$ (iii) the conditional distribution of Y given $X = 2$.