V.P.& R.P.T.P.Science College , Vallabh Vidyanagar.
B.Sc. (Semester - III) Internal Test
US03CMTH22 (Multivariate Calculus)
Date. 9/10/2019 ; Wednesday 3:00 p.m. to 4:15 p.m. Maximum Marks: 25

Que.1 Fill in the blanks. 5

(1) The value of $\int_0^\infty e^{-x}\cos 2x\, dx =$ .....
(a) 0 (b) $-1/5$ (c) $1/5$ (d) $2/5$

(2) For $x+y=u$, $x-2y=v$, jacobian $|J| =$ .....
(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

(3) For the curve $y=-x$, $\frac{ds}{dt} =$ .....
(a) 2 (b) 1 (c) $\sqrt{2}$ (d) $-\sqrt{2}$

(4) If $\bar{r} = u\,\bar{i} + v\,\bar{j} + uv\,\bar{k}$ then $\bar{r}_u \cdot \bar{r}_v =$ .....
(a) $1+v^2$ (b) uv (c) $1+v^2+u^2$ (d) $1+u^2$

(5) $\int_0^1\int_0^1\int_0^1 xyz\, dxdydz =$ .....
(a) 8 (b) 1/8 (c) 1/2 (d) 1/4

Que.2 (a) State and prove relation between Beta and Gamma function. 5

OR

Que.2 (b) Prove that $\text{curl}(\mathbf{U}\times\mathbf{V}) = \mathbf{U}\,\text{div}\,\mathbf{V} - \mathbf{V}\,\text{div}\,\mathbf{U} + (\mathbf{V}\cdot\nabla)\mathbf{U} - (\mathbf{U}\cdot\nabla)\mathbf{V}$. 5

Que.3 (a) Find volume of the Region bounded by paraboloid $x^2+4y^2 = z$, plane $z=0$ and by the cylinder $y^2 = x$ and $x^2 = y$. 5

OR

Que.3 (b) Transform $\iint_R (x-y)^2\sin^2(x+y)\,dxdy$ in uv-plane by taking $x-y=u$, $x+y=v$ .Then evaluate it, where R : Parallelogram with vertices $(\pi,0),(2\pi,\pi),(\pi,2\pi),(0,\pi)$. 5

Que.4 (a) State and prove Green's theorem for plane . 5

OR

Que.4 (b) Evaluate $\iint_S f(x,y,z)\,dA$. for $f(x,y,z) = \tan^{-1}(y/x)$, S : $z = x^2+y^2$, $1\le z\le 4$, $x\ge 0$, $y\ge 0$. 5

Que.5 (a) Verify Stoke's theorem for $\bar{V} = y^3\bar{i} - x^3\bar{j}$ and surface S : the circular disk $x^2+y^2\le 1, z=0$. 5

OR

Que.5 (b) State and prove Stoke's theorem. 5

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