V.P. AND R.P.T.P. SCIENCE COLLEGE
INTERNAL EXAMINATION
B.Sc.SEMESTER -IV
SUB: Mathematics (US04EMTH05)
(CALCULUS AND ALGEBRA - II)

Date : 13/03/2019 **Maximum Marks : 50**
Day : Wednesday **Time : 3:00 pm to 5:00 pm**

Q.1 Attempt the following. 8

(1) The solution of Laplace's equation is called .... function .

(a) Constant (b) Continuous (c) Harmonic (d) Laplacian operator

(2) The divergent of vector field $\overline{v} = x^3\overline{i} + 4y^2\overline{j}$ is........

(a) $3x^2 + 8y$ (b) $2x^2 - 2y$ (c) 0 (d) $2x$

(3) For Boolean algebra B, $a + a =$ ....

(a) $2a$ (b) 0 (c) 1 (d) a

(4) Find A in $AC - B^2$ for $x^3 + y^3 - 63(x + y) + 12xy$ at the point (3,3)

(a) 6 (b) 14 (c) 12 (d) 18

(5) $\overline{\nabla} \cdot (f\overline{\nabla}g) =$ ....

(a)$f\overline{\nabla}^2 g + \overline{\nabla} f \cdot \overline{\nabla}$ (b) $g\overline{\nabla}^2 f - \overline{\nabla} f \cdot \overline{\nabla} g$ (c) $f\overline{\nabla}^2 g - \overline{\nabla} f \cdot \overline{\nabla} g$ (d) none

(6) $\overline{\nabla}(8f + 6g)$ ....

(a) $8\overline{\nabla} f + 6\overline{\nabla} g$ (b) $8\overline{\nabla} f$ (c) $8\overline{\nabla} f - 6\overline{\nabla} g$ (d) 0

(7) For Boolean algebra B, $p \wedge 1 =$ ....

(a) 1 (b) p (c) 0 (d) $p \vee 1$

(8) If $f(x, y) = x^3 + y^3 - 63x - 63y + 12xy$ then $f_{yy} =$ ....

(a) $6y$ (b) $63y$ (c) $3y^2$ (d) none

Q.2 Attempt the following (Any five). 10

(1) Show that $(y - x)^4 + (x - 2)^4$ has minimum at (2,2)

(2) Define global maxima and local minima.

(3) Prove that $\overline{\nabla}.(\overline{\nabla} f) = \overline{\nabla}^2 f$.

(4) Find $\overline{\nabla} \cdot \left(\frac{\overline{r}}{r^3}\right)$, $where\ \overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$

(5) Define Boolean Algebra.

(6) Find the gradient of the function $f(x, y) = (x^2 + y^2 + z^2)^2$ at (1 , 2 , 3).

(7) Prove that $\overline{\nabla}(fg) = f\overline{\nabla}g + g\overline{\nabla}f$.

(8) Prove that (i) $a + a = a$ (ii) $a + 1 = 1$.

Q.3 [A] A rectangular box open at the top is to have a volume of $32m^3$. Find the dimension of box so that the total surface area is minimum. 5

Q.3 [B] Find stationary points for the function $f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$. 3

OR

Q.3 [C] Show that $2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum at $(0,0)$. 4

Q.3 [D] Find the local maxima and minima of, $x^4 - 2x^2 - 2y^2 + 4xy + y^4$. 4

Q.4 [A] Prove that $\tan^{-1}\left(\frac{x}{y}\right)$ is harmonic function. 4

Q.4 [B] Find unit normal vector of the given surface $z^2 = x^2 + y^2$ at point $(3,4,5)$. 4

OR

Q.4 [C] Find directional derivative of $f(x,y,z) = 4xz^3 - 3x^2 + y^2z$ at point (2 ,-1 ,2) in the direction $\overline{a} = 2\overline{i} - 3\overline{j} + 6\overline{k}$ 4

Q.4 [D] Find the gradient of the function $f(x,y) = \dfrac{x}{x^2+y^2}$ at $(2,3)$ 4

Q.5 [A] Find $\overline{\nabla} \cdot (r^n \overline{r})$, $where$ $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$ 4

Q.5 [B]Verify $\overline{\nabla} \cdot (f\overline{\nabla}g) = f\overline{\nabla}^2 g + \overline{\nabla}f \cdot \overline{\nabla}g$ for $f = x + y + z$, $g = xyz$. 4

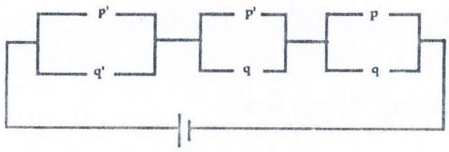
OR

Q.5 [C] If $f(x,y) = \log(x^2 + y^2)$ then prove that $\nabla^2 f = 0$ 4

Q.5 [D] Prove that $\overline{\nabla} \cdot (\overline{\nabla} \times \overline{V}) = 0$ 4

Q.6 [A] Prove that in Boolean algebra B, binary operation is associative. 4

Q.6 [B] Find Boolean function of given switching circuit and simplify it. 4



OR

Q.6 [C] State and prove De-Morgen's laws for Boolean algebra B 5

Q.6 [D] If $a + x = b + x$ & $a + x' = b + x'$ then prove that $a = b$. 3

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