V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2018-19

Subject : Mathematics US04CMTH02 Max. Marks : 50

Differential Equations

Date: 12/03/2019 Timing: 03:00 pm - 05:00 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. 8

[1] Integral curve of $dx = 4ydy = 6zdz$ is given by

[A] $x + 2y^2 = c_1, x + 3z^2 = c_2$ [B] $x^2 + 2y^2 = c_1, x^2 + 3z^2 = c_2$

[C] $x^2 = 2y^2 + c_1, x^2 = 3z^2 + c_2$ [D] $x = 2y^2 + c_1, x = 3z^2 + c_2$

[2] Integral curve of $e^x dx = e^y dy = e^z dz$ is given by

[A] $e^x + e^y = c_1$; $e^y - e^z = c_2$ [B] $e^x - e^y = c_1$; $e^y + e^z = c_2$

[C] $e^x + e^y = c_1$; $e^y + e^z = c_2$ [D] $e^x - e^y = c_1$; $e^y - e^z = c_2$

[3] $ax + by + z = 5$ is a solution of

[A] px-qy+z=5 [B] qx-py+z=5 [C] px+qy-z=-5 [D] none

[4] The general solution of the partial differential equation $p+q = 1$ is an arbitrary function $F(u, v) = 0$, where $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ are solutions of

[A] $dx + dy + dz = 0$ [B] $dx + dy = dz$ [C] $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ [D] $dx = dy = dz$

[5] The surfaces orthogonal to a one parameter family of surfaces $2x^2+3y^2+4z^2 = c$ are the surfaces generated by the integral curves of the equations

[A] $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ [B] $\frac{dx}{2x} = \frac{dy}{3y} = \frac{dz}{4z}$

[C] $2xdx = 3ydy = 4zdz$ [D] $xdx = ydy = zdz$

[6] Integral surface of the linear partial differential equation $x^2p - y^2q = z^2$ can be obtained by solving the differential equation

[A] $\frac{dx}{z^2} = -\frac{dy}{x^2} = \frac{dz}{y^2}$ [B] $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2}$

[C] $\frac{dx}{y^2} = -\frac{dy}{z^2} = \frac{dz}{x^2}$ [D] $\frac{dx}{x^2} = -\frac{dy}{y^2} = \frac{dz}{z^2}$

[7] The complete integral of $z = px + qy + p^2 - q^2$ is given by ____ , where a and b are arbitrary constants.

[A] $z = ax + by$ [B] $z = ay + bx + a^2 - b^2$

[C] $z = a^2x - b^2y$ [D] $z = ax + by + a^2 - b^2$

[8] For a partial differential equation $pq = 5$, the Charpit's auxiliary equations are given by

[A] $\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{2pq} = \frac{dp}{0} = \frac{dq}{0}$ [B] $\frac{dx}{p} = \frac{dy}{q} = \frac{dz}{2pq} = \frac{dp}{0} = \frac{dq}{0}$

[C] $\frac{dx}{q} = \frac{dy}{p} = \frac{dz}{2pq} = \frac{dp}{0} = \frac{dq}{0}$ [D] $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{2pq} = \frac{dp}{0} = \frac{dq}{0}$

Q: 2. Answer any FIVE of the following. 10

[1] Find the integral curves of the equations $\frac{dx}{\sqrt{x}} = \frac{dy}{\sqrt[3]{y}} = \frac{dz}{\sqrt[4]{z}}$

[2] Find the integral curves of $\frac{dx}{2} = -\frac{dy}{3} = \frac{dz}{4}$

[3] Determine whether the equation $ydx + xdy = 5zdz$ is integrable or not.

[4] Obtain partial differential equation of $ax - by + 4z = 12$

[5] Obtain a differential equation of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ whose solution generates surfaces orthogonal to the surfaces $5x^2 + 6y^2 + 7z^2 = c$

[6] Find a differential equation which can be solved to obtain integral curve of the linear partial differential equation $px - qy^2 = z^2$

[7] Find the Charpit's auxiliary equations for $5p^2q^2 = 1$

[8] Find the general solution of $(D - 3D')z = 0$.

Q: 3 [A] Find the integral curves of the equations

$$\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$$

5

[B] Solve : $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$ 3

OR

Q: 3 [A] Solve : $\frac{dx}{z} = \frac{dy}{-z} = \frac{dz}{z^2 + (x+y)^2}$ 5

[B] Solve : $\frac{dx}{y^2(x-y)} = \frac{dy}{-x^2(x-y)} = \frac{dz}{z(x^2+y^2)}$ 3

Q: 4 [A] If X is a vector such that $X.curlX = 0$ and μ is an arbitrary function of x, y and z then prove that $(\mu X).curl(\mu X) = 0$ 5

[B] Determine whether the Pfaffian differential equation

$$(y+z)dx + (z+x)dy + (x+y)dz = 0$$

is integrable or not. Find its solution if it is integrable 3

OR

Q: 4 [A] Prove that a necessary and sufficient condition that the Pfaffian differential equation $X.dr = 0$ is integrable is that $X.curlX = 0$ 5

[B] Determine whether the Pfaffian differential equation $z(z+y)dx+z(z+x)dy-2xydz=0$ is integrable or not. Find its solution if it is integrable **3**

Q: 5 [A] Find Integral Surface of the linear partial differential equation $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ which contains the straight line $x+y=0;\ z=1$ **4**

[B] Find the surface which is orthogonal to one parameter system $z=cxy(x^2+y^2)$ and which passes through the hyperbolas $x^2-y^2=a^2;\ z=0$ **4**

OR

Q: 5 [A] Find the integral surface of the linear partial differential equation $2y(z-3)p+(2x-z)q=y(2x-3)$ passing through the circle $z=0, x^2+y^2=2x$ **4**

[B] Find the integral surface of the equation $x^2p+y^2q=-z^2$ which passes through the hyperbola $xy=x+y,\ \ z=1$ **4**

Q: 6 [A] Prove that two first order partial differential equations $f(x,y,z,p,q)=0$ and $g(x,y,z,p,q)=0$ are compatible if $[f,g]=0$ where

$$[f,g]=\frac{\partial(f,g)}{\partial(x,p)}+p\frac{\partial(f,g)}{\partial(z,p)}+\frac{\partial(f,g)}{\partial(y,q)}+q\frac{\partial(f,g)}{\partial(z,q)}$$

5

[B] Find the complete integral of $p+q=pq$ **3**

OR

Q: 6 [A] Show that the equations $xp-yq=x$ and $x^2p+q=xz$ are compatible and find their solution. **5**

[B] If $\mu_1,\mu_2,\dots,\mu_n$ are solutions of homogeneous linear differential equation $F(D,D')z=0$ then $\sum_{r=1}^{n}c_r\mu_r$ is also solution of $F(D,D')z=0$, where c_r are arbitrary constants **3**