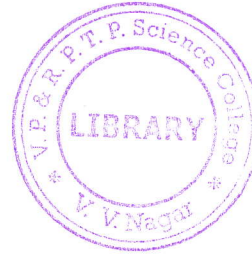


V.P. AND R.P.T.P. SCIENCE COLLEGE  
 B.Sc.SEMESTER -III  
 INTERNAL EXAMINATION  
 SUBJECT :MATHEMATICS (CALCULUS AND ALGEBRA - I)  
 SUBJECT CODE : US03EMTH05

Date : 8/10/2018  
 Day : Monday

Maximum Marks : 50  
 Time :3 p.m. to 5 p.m.

Que.1 Attempt the following.



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- (1)  $\lim_{x \rightarrow 2} \frac{\sin x}{x} = \dots\dots$   
 (a) 0 (b)  $-\infty$  (c) 1 (d) -1
- (2)  $\lim_{x \rightarrow 0} \frac{\log(\cos x)}{\sin x}$  is of the form .....  
 (a)  $\infty - \infty$  (b)  $\frac{\infty}{\infty}$  (c)  $\infty^0$  (d)  $\frac{0}{0}$
- (3) If  $f = \frac{x^2 + y^2}{\sqrt{x} + \sqrt{y}}$  is homogeneous function of degree .....  
 (a) 2 (b)  $\frac{1}{2}$  (c)  $\frac{3}{2}$  (d) 1
- (4) If  $f = \sin x$  then  $f_{xx} = \dots\dots\dots$   
 (a)  $\sin x$  (b)  $-\cos x$  (c)  $\cos x$  (d)  $-\sin x$
- (5) Unit matrix is also known as .....  
 (a) Identity matrix (b) Null matrix (c) skew symmetry (d) Hermitian
- (6) If  $A = \begin{pmatrix} 1+2i & 3 \\ 8 & 5+6i \end{pmatrix}$  then conjugate of A is.....  
 (a)  $\begin{pmatrix} 1-2i & 3 \\ 8 & 5+6i \end{pmatrix}$  (b)  $\begin{pmatrix} 1+2i & -3 \\ -8 & 5+6i \end{pmatrix}$  (c)  $\begin{pmatrix} 1-2i & 3 \\ 8 & 5-6i \end{pmatrix}$  (d)  $\begin{pmatrix} 1+2i & 3 \\ 8 & 5-6i \end{pmatrix}$
- (7) If  $A = \begin{pmatrix} 2 & 4 \\ 1 & -2 \end{pmatrix}$  then determinant of A is .....  
 (a) 8 (b) -8 (c) -4 (d) 4
- (8) If  $Y = \begin{pmatrix} 2 & i \\ 1 & 4 \end{pmatrix}$ ,  $Z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  then  $Y + Z$  is .....  
 (a)  $\begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$  (b)  $\begin{pmatrix} 3 & i \\ 1 & 5 \end{pmatrix}$  (c)  $\begin{pmatrix} 2 & i \\ 1 & 5 \end{pmatrix}$  (d)  $\begin{pmatrix} 3 & i \\ 1 & 4 \end{pmatrix}$

Que.2 Attempt the following.(any five)

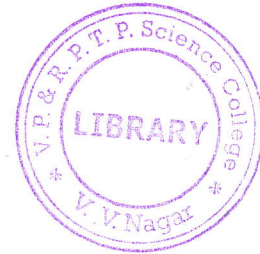
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- (1) Evaluate  $\lim_{x \rightarrow 0} \frac{\log(\sin 2x)}{\log(\sin x)}$
- (2) Evaluate  $\lim_{x \rightarrow 2} \frac{\sin(x^2 - 4)}{x - 2}$
- (3) Define Homogeneous function with a example.
- (4) Prove that  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  for  $u = 2(ax + by)^2 - x^2 - y^2$  where  $a^2 + b^2 = 1$
- (5) If A is Hermitian then prove that  $B^{\theta}AB$  is Hermitian.
- (6) Define triangular matrix and identity matrix with example.
- (7) Define Determinant and Minor of matrix with example.
- (8) If  $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$  then find characteristic matrix and characteristic equation of A.

Que.3 [A] Evaluate  $\lim_{x \rightarrow 0} \frac{e^x + \log(1-x) - 1}{\tan x - x}$  4

[B] Evaluate  $\lim_{x \rightarrow 1} (4 - 4x^2)^{\log(2-2x)}$  4

OR



Que.3 [C] Evaluate  $\lim_{x \rightarrow 0} \left[ \frac{\tan x}{x} \right]^{\frac{5}{3x^2}}$  4

[D] Evaluate  $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2 \log(1+x)}{x \sin x}$  4

Que.4 [A] For  $u = x^3 - 3xy^2$  prove that  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ , Also prove that  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$  4

[B] If  $H = f(2x - 3y, 3y - 4z, 4z - 2x)$  then prove that  $\frac{1}{2} \frac{\partial H}{\partial x} + \frac{1}{3} \frac{\partial H}{\partial y} + \frac{1}{4} \frac{\partial H}{\partial z} = 0$  4

OR

Que.4 [C] State and prove Euler's theorem for two variables. 4

[D] If  $u = \sqrt{x^2 + y^2}$  then find  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$  4

Que.5 [A] For  $A = \begin{pmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{pmatrix}$ , where  $l = \frac{1}{\sqrt{2}}, m = \frac{1}{\sqrt{6}}, n = \frac{1}{\sqrt{3}}$ , show that  $AA' = I$  4

[B] Prove that Every square matrix can be expressed in one and only one way as  $P + iQ$  where  $P$  and  $Q$  are Hermitian matrices. 4

OR

Que.5 [C] If  $A = \begin{pmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{pmatrix}$  and  $2X + 3A = B$  then find  $X$ . 4

[D] If  $A$  and  $B$  are both symmetric then prove that  $AB$  is symmetric iff  $A$  and  $B$  commute. 4

Que.6 [A] State and prove Cayley-Hamilton theorem. 6

[B] If  $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$  then prove that  $A^2 - 4A + 5I = 0$  2

OR

Que.6 [C] Verify Cayley-Hamilton theorem for  $A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$  4

[D] If  $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  and  $E = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$  then prove that  $(aI + bE)^3 = a^3I + 3a^2bE$  4

