V.P. AND R.P.T.P. SCIENCE COLLEGE
B.Sc.SEMESTER -III
INTERNAL EXAMINATION
SUBJECT :MATHEMATICS (CALCULUS AND ALGEBRA - I)
SUBJECT CODE : US03EMTH05

Date : 8/10/2018 **Maximum Marks : 50**
Day : Monday **Time :3 p.m. to 5 p.m.**

Que.1 Attempt the following. 8

(1) $\lim_{x\to 2} \frac{\sin x}{x} = \ldots\ldots\ldots$
(a) 0 (b) $-\infty$ (c) 1 (d) -1

(2) $\lim_{x\to 0} \frac{\log(\cos x)}{\sin x}$ is of the form ....
(a) $\infty - \infty$ (b) $\frac{\infty}{\infty}$ (c) ∞^0 (d) $\frac{0}{0}$

(3) If $f = \frac{x^2 + y^2}{\sqrt{x} + \sqrt{y}}$ is homogeneous function of degree ....
(a) 2 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 1

(4) If $f = \sin x$ then $f_{xx} = \ldots\ldots\ldots\ldots$
(a) $\sin x$ (b) $-\cos x$ (c) $\cos x$ (d) $-\sin x$

(5) Unit matrix is also known as ....
(a) Identity matrix (b) Null matrix (c) skew symmetry (d) Hermitian

(6) If $A = \begin{pmatrix} 1+2i & 3 \\ 8 & 5+6i \end{pmatrix}$ then conjugate of A is.......
(a) $\begin{pmatrix} 1-2i & 3 \\ 8 & 5+6i \end{pmatrix}$ (b) $\begin{pmatrix} 1+2i & -3 \\ -8 & 5+6i \end{pmatrix}$ (c) $\begin{pmatrix} 1-2i & 3 \\ 8 & 5-6i \end{pmatrix}$ (d) $\begin{pmatrix} 1+2i & 3 \\ 8 & 5-6i \end{pmatrix}$

(7) If $A = \begin{pmatrix} 2 & 4 \\ 1 & -2 \end{pmatrix}$ then determinant of A is ....
(a) 8 (b) -8 (c) -4 (d) 4

(8) If $Y = \begin{pmatrix} 2 & i \\ 1 & 4 \end{pmatrix}$, $Z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then $Y + Z$ is ....
(a) $\begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & i \\ 1 & 5 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & i \\ 1 & 5 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & i \\ 1 & 4 \end{pmatrix}$

Que.2 Attempt the following.(**any five**) 10

(1) Evaluate $\lim_{x\to 0} \frac{\log(\sin 2x)}{\log(\sin x)}$
(2) Evaluate $\lim_{x\to 2} \frac{\sin(x^2-4)}{x-2}$
(3) Define Homogeneous function with a example.
(4) Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for $u = 2(ax+by)^2 - x^2 - y^2$ where $a^2 + b^2 = 1$
(5) If A is Hermitian then prove that $B^\theta AB$ is Hermitian.
(6) Define triangular matrix and identity matrix with example.
(7) Define Determinant and Minor of matrix with example.
(8) If $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ then find characteristic matrix and characteristic equation of A.

Que.3 [A] Evaluate $\lim_{x\to 0} \dfrac{e^x + \log(1-x) - 1}{\tan x - x}$ 4

[B] Evaluate $\lim_{x\to 1} (4-4x^2)^{\frac{1}{\log(2-2x)}}$ 4

OR

Que.3 [C] Evaluate $\lim_{x\to 0} \left[\dfrac{\tan x}{x}\right]^{\frac{5}{3x^2}}$ 4

[D] Evaluate $\lim_{x\to 0} \dfrac{e^x - e^{-x} - 2\log(1+x)}{x\sin x}$ 4

Que.4 [A] For $u = x^3 - 3xy^2$ prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$, Also prove that $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$ 4

[B] If $H = f(2x-3y, 3y-4z, 4z-2x)$ then prove that $\dfrac{1}{2}\dfrac{\partial H}{\partial x} + \dfrac{1}{3}\dfrac{\partial H}{\partial y} + \dfrac{1}{4}\dfrac{\partial H}{\partial z} = 0$ 4

OR

Que.4 [C] State and prove Euler's theorem for two variables. 4

[D] If $u = \sqrt{x^2+y^2}$ then find $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}$ 4

Que.5 [A] For $A = \begin{pmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{pmatrix}$, where $l = \frac{1}{\sqrt{2}}, m = \frac{1}{\sqrt{6}}, n = \frac{1}{\sqrt{3}}$, show that $AA' = I$ 4

[B] Prove that Every square matrix can be expressed in one and only one way as $P + iQ$ where P and Q are Hermitian matrices. 4

OR

Que.5 [C] If $A = \begin{pmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{pmatrix}$ and $2X + 3A = B$ then find X. 4

[D] If A and B are both symmetric then prove that AB is symmetric iff A and B commute. 4

Que.6 [A] State and prove Cayley-Hamilton theorem. 6

[B] If $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ then prove that $A^2 - 4A + 5I = 0$ 2

OR

Que.6 [C] Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$ 4

[D] If $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $E = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ then prove that $(aI + bE)^3 = a^3 I + 3a^2 bE$ 4

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