V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2015-16

Subject : Mathematics US04CMTH02 Max. Marks : 25

Differential Equations

Date: 18/03/2016 Timing: 3.00 pm - 04.30 pm

Instructions : (1) This question paper contains FIVE questions.
(2) The figures to the right side indicate full marks of the corresponding question/s
(3) The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. 3

[1] Integral curve of $ax^2dx = by^2dy = cz^2dz$ is given by

[A] $ax^3 - by^3 = c_1, by^3 - cz^3 = c_2$ [B] $ax^3 + by^3 = c_1, by^3 + cz^3 = c_2$

[C] $ax^3 + by^3 = c_1, by^3 - cz^3 = c_2$ [D] $ax^3 - by^3 = c_1, by^3 + cz^3 = c_2$

[2] A Paffian differential equation in two variables

[A] has no solution

[B] can have solution if certain conditions are satisfied

[C] always possesses a solution

[D] none of these

[3] Intersection of two different surfaces in R^3 is

[A] a curve in R^3

[B] is a curve in R^2

[C] an orthogonal surface to both the surfaces

[D] none

Q: 2. Answer any TWO of the following. 4

[1] Find the integral curves of $\frac{dx}{x^2} = \frac{2dy}{y^2} = \frac{dz}{z^2}$

[2] Obtain partial differential equation of $z = ax + by$

[3] Obtain a differential equation of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dy}{R}$ whose integral curves generate surfaces orthogonal to the surfaces $x^2 - 5y^2 - 2z^2 = c$

Q: 3 [A] Solve : $\frac{dx}{1+x} = \frac{dy}{1+y} = \frac{dz}{z}$ 3

[B] Solve : $\frac{dx}{2xz} = \frac{dy}{2yz} = \frac{dz}{z^2 - x^2 - y^2}$ 3

OR

Q: 3. Find the orthogonal trajectories on the surface $x^2 + y^2 + 2fyz + d = 0$ of the fix curves intersecting with planes parallel to the plane XOY 6

Q: 4 [A] Prove that a necessary and sufficient condition that there exists, between two functions $u(x,y)$ and $v(x,y)$ a relation $F(u,v) = 0$ not involving x and y explicitly is that $\dfrac{\partial(u,v)}{\partial(x,y)} = 0$ 3

[B] Solve : $x^2\dfrac{\partial z}{\partial x} + y^2\dfrac{\partial z}{\partial y} = (x+y)z$ 3

OR

Q: 4 [A] Prove that the general solution of the linear differential equation $pP+qQ = R$ is $F(u,v) = 0$, where $F(u,v) = 0$ is an arbitrary function of $u(x,y,z) = c_1$ and $v(x,y,z) = c_2$ which form a solution of the equation $\dfrac{dx}{P} = \dfrac{dy}{Q} = \dfrac{dz}{R}$ 3

[B] Define Paffian differential equation and find the solution of Determine whether the Pfaffian differential equation $(x^2z-y^3)dx+3xy^2dy+x^3dz = 0$ is integrable or not. Find its primitive if it is integrable 3

Q: 5. Find the integral surface of the equation $(x-y)y^2p+(y-x)x^2q = (x^2+y^2)z$ through the curve $xz = a^3, y = 0$ 6

OR

Q: 5. Find the surface which is orthogonal to the surface $z(x+y) = c(3z+1)$ and which passes through the circle $x^2+y^2 = 1,\ z = 1$ 6