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V.P.& R.P.T.P.Science College , Vallabh Vidyanagar.
B.Sc.(Semester - I) Internal Test
US01CMTH21 (CALCULUS)

Date. 7/10/2019 ; Monday 1.00 p.m. to 2.15 p.m. Maximum Marks: 25

Que.1 Fill in the blanks. 8

(1) $\cosh x - \sinh x =$ ....
(a) 1 (b) e^{x} (c) e^{-x} (d) -1

(2) The n^{th} derivative of the function $e^{3x}\ \cos 4x$ is ....
(a) $7^{n}e^{3x}\ \cos 4x$ (b) $5^{n}e^{3x}\ \cos(4x + n\tan^{-1}\frac{4}{3})$ (c) $e^{3x}\ \cos(4x + n\frac{\pi}{2})$ (d) None

(3) The curve of $r = a\theta$ is is symmetric about ....
(a) polar axis (b) normal axis (c) pole (d) polar axis , normal axis and pole

(4) $\int_{0}^{\pi/2} \sin^{10} x\, dx =$ ....
(a) $\frac{63}{265}$ (b) $\frac{63}{512}$ (c) $\frac{63\pi}{512}$ (d) None

(5) If $\overline{r(t)}$ is differentiable vector function of constant length then ....
(a) $\bar{r} \times \frac{d\,\bar{r}}{dt} = \bar{0}$ (b) $\bar{r}\frac{d\,\bar{r}}{dt} = 0$ (c) $\bar{r} \cdot \frac{d\,\bar{r}}{dt} = \bar{0}$ (d) $\frac{d\,\bar{r}}{dt} \cdot \bar{r} = 0$

Que.2 (a) State and prove Leibniz's theorem . Hence find y_n for $y = x log(x - 1)$. 5

OR

Que.2 (b) Evaluate $\lim_{x\to 0} \frac{e^{x} + \log(1-x) - 1}{\tan x - x}$. 5

Que.3 (a) Sketch the curve given by $y = \frac{(x-1)(x+3)}{x(x+2)}$. 5

OR

Que.3 (b) In usual notation prove that $r = \frac{p\,e}{1 \pm e\ cos\theta}$. 5

Que.4 (a) Prove that the length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ measured from $(0, a)$ to the point (x, y) is given by $\frac{3}{2}(ax^{2})^{1/3}$. 5

OR

Que.4 (b) Obtain Reduction Formula for $\int \sin^{n} x\, dx$ and $\int_{0}^{\pi/2} \sin^{n} x\, dx$ where $n \in \mathbb{N}$. 5

Que.5 (a) Prove that if ρ is the radius of curvature at any point P of the parabola $y^{2} = 4ax$ and S is its focus then prove that $\rho^{2} \propto SP^{3}$. 5

OR

Que.5 (b) State and prove Euler's theorem for homogeneous function $z = f(x, y)$ of degree n . If all the second order partial derivatives of f exist and are continuous , then prove that $x^{2}\frac{\partial^{2} z}{\partial x^{2}} + 2xy\frac{\partial^{2} z}{\partial x \partial y} + y^{2}\frac{\partial^{2} z}{\partial y^{2}} = n(n-1)z$. 5

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