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V.P.& R.P.T.P.Science College , Vallabh Vidyanagar.
B.Sc.(Semester - I) Internal Test
US01CMTH21 (CALCULUS)

Date. 5/10/2017 ; Friday 12.30 p.m. to 2.30 p.m. Maximum Marks: 50

Que.1 Fill in the blanks. 8

(1) $\cosh x - \sinh x =$ ....
(a) 1 (b) e^x (c) e^{-x} (d) -1

(2) The n^{th} derivative of the function $e^{3x}\ \cos 4x$ is ....
(a) $7^n e^{3x}\ \cos 4x$ (b) $5^n e^{3x}\ \cos(4x + n\tan^{-1}\frac{4}{3})$ (c) $e^{3x}\ \cos(4x + n\frac{\pi}{2})$ (d) None

(3) Parametric equation for $x^{2/3} - y^{2/3} = a^{2/3}$ are ....

(a) $x = a\cos^3\theta$; $y = a\sin^3\theta$ (b) $x = a\sec^3\theta$; $y = a\tan^3\theta$

(c) $x = \cos^3\theta$; $y = \sin^3\theta$ (d) $x = a\tan^3\theta$; $y = a\sec^3\theta$

(4) The curve of $r = a\theta$ is is symmetric about ....
(a) polar axis (b) normal axis (c) pole (d) polar axis , normal axis and pole

(5) $\int_0^{\pi/2} \sin^{10} x\, dx =$ .... $n \in \mathbb{N}$.
(a) $\frac{63}{265}$ (b) $\frac{63}{512}$ (c) $\frac{63\pi}{512}$ (d) None

(6) Volume by Cylindrical cell method is V = ...
(a) $2\pi\int_a^b xy\,dx$ (b) $\pi\int_a^b xy\,dx$ (c) $\pi\int_a^b x^2\,dx$ (d) None

(7) If $\overline{r(t)}$ is differentiable vector function of constant length then ....
(a) $\overline{r} \times \frac{d\,\overline{r}}{dt} = \overline{0}$ (b) $\overline{r}\frac{d\,\overline{r}}{dt} = 0$ (c) $\overline{r} \cdot \frac{d\,\overline{r}}{dt} = \overline{0}$ (d) $\frac{d\,\overline{r}}{dt} \cdot \overline{r} = 0$

(8) .... has infinite radius of curvature at any point .
(a) Circle with radius 4 (b) Parabola $y^2 = 4ax$ (c) Line $y = x$ (d) None

Que.2 Answer the following (Any Five) 10

(1) Evaluate $\int \frac{dx}{\sqrt{4x^2 - 9}}$

(2) For an integer m if $y = (ax+b)^m$, then prove that $y_n = m(m-1)\cdots(m-n+1)a^n(ax+b)^{m-n}$.

(3) Find any one oblique asymptote for the curve given by $x = t + \frac{1}{t^2}$; $y = t - \frac{1}{t^2}$.

(4) Find Tangent parallel to the axes and Extent for $x = cos^2\theta$; $y = 2sin\theta$.

(5) Evaluate $\int_0^{\pi/4} \cos^3 2x\ \sin^4 4x\, dx$.

(6) Evaluate $\int_0^{\pi} \frac{\sin^4\theta}{(1+\cos\theta)^2}\, d\theta$.

(7) If $u = \sin^{-1}(\frac{x^2y^2}{x+y})$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$.

(8) Find $\frac{dy}{dx}$ for $x\sin(x-y) - (x+y) = 0$.

Que.3 (a) State and prove Leibniz's theorem . Hence find y_n for $y = xlog(x-1)$. 5

(b) In usual notation prove that $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$; $x \geq 1$ 3

OR

Que.3 (c) Evaluate $\lim_{x \to 0} \dfrac{e^x + \log(1-x) - 1}{\tan x - x}$.

(d) Find center-to-focus distance , foci and asymptotes for the hyperbola $\dfrac{x^2}{4} - \dfrac{y^2}{5} = 1$. 3

Que.4 (a) Sketch the curve given by $y = \dfrac{(x-1)(x+2)}{x(x-4)}$. 4

(b) In usual notation prove that $r = \dfrac{p\,e}{1 \pm e\,cos\theta}$. 4

OR

Que.4 (c) If a curve is given by $x = f(t)$; $y = g(t)$ and that both x and y get numerically large as t approaches some number , say a . Then an oblique asymptote to the curve ,if it exist, is given by $y = mx + c$, where $m = \lim_{t \to a} \dfrac{dy}{dx}$ and $c = \lim_{t \to a} (y - mx)$. 4

(d) Sketch the curve given by $r = 2 - \cos\theta$. 4

Que.5 (a) Prove that the length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ measured from $(0, a)$ to the point (x, y) is given by $\frac{3}{2}(ax^2)^{1/3}$. 5

(b) Evaluate $\int_0^1 x^5 \sqrt{\frac{1+x^2}{1-x^2}}\, dx$. 3

OR

Que.5 (c) Obtain Reduction Formula for $\int \sin^n x\, dx$ where $n \in \mathbb{N}$. 4

(d) The area bounded by the parabola $x^2 = 8y$ and the line $y = 2$ is revolved about the line $y = 2$. Find the volume of the solid thus generated . 4

Que.6 (a) Prove that if ρ is the radius of curvature at any point P of the parabola $y^2 = 4ax$ and S is its focus then prove that $\rho^2 \propto SP^3$. 5

(b) For $\bar{r}(t) = 3\cos t\, \bar{i} + 3\sin t\, \bar{j} + t^2\, \bar{k}$. Find 3
(i) the velocity vector and acceleration vector (ii) the speed at any time t (iii) the time , if any , when the acceleration is orthogonal to velocity .

OR

Que.6 (c) State and prove Euler's theorem for homogeneous function $z = f(x, y)$ of degree n . If all the second order partial derivatives of f exist and are continuous , then prove that
$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z$. 5

(d) If A , B and C are angles of a ΔABC such that $\sin^2 A + \sin^2 B + \sin^2 C = K$, a constant, then prove that $\frac{dB}{dC} = \frac{\tan C - \tan A}{\tan A - \tan B}$. 3

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