V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2015-16

Subject : Mathematics US01CMTH02 Max. Marks : 25

Calculus and Differential Equations

Date: 09/10/2015 Timing: 01:30 pm - 02:30 pm

Instructions : (1) This question paper contains 5 questions.
(2) The figures to the right side indicate full marks of the corresponding question/s
(3) The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[1] If $y = \sin 3x$ then $y_{10} =$

[A] $3^{10} \sin 3x$ [B] $-3^{10} \sin 3x$ [C] $3^{10} \cos 3x$ [D] $-3^{10} \cos 3x$

[2] For $r = f(\theta)$ which of the following is not true ?

[A] $\frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ [B] $\tan\phi = \frac{r}{r_1}$

[C] $\rho = \frac{(r^2 + {r_1}^2)^{\frac{3}{2}}}{r^2 + 2{r_1}^2 - rr_2}$ [D] $\frac{ds}{d\theta} = \sqrt{1 + \left(\frac{dr}{d\theta}\right)^2}$

[3] If a function y of x be implicitly described by $f(x, y) = c$, where c is a constant then

[A] $\frac{dy}{dx} = -\frac{f_y}{f_x}$ [B] $\frac{dy}{dx} = \frac{f_y}{f_x}$ [C] $\frac{dy}{dx} = \frac{f_x}{f_y}$ [D] $\frac{dy}{dx} = -\frac{f_x}{f_y}$

Q: 2. Answer any TWO of the following. **4**

[1] If $y = \log(2x - 1)$ then find y_4

[2] Define : (i) Average Curvature (ii) Intrinsic Equation

[3] Determine whether $f(x, y) = \frac{\sqrt[4]{x} - \sqrt[4]{y}}{x^2 - y^2}$ is a homogeneous function or not.

Q: 3 [A] Find y_n for $y = e^{2x} \cos x \sin^2 2x$ **3**

[B] If $y = e^{ax}\cos(bx + c)$, then prove that $y_n = r^n e^{ax} \cos(bx + c + n\varphi)$, where $r = \sqrt{a^2 + b^2}$, $\varphi = \tan^{-1}\left(\frac{b}{a}\right)$ **3**

OR

Q: 3. If $y = (x - \sqrt{4 + x^2})^m$,then find $y_n(0)$ **6**

Q: 4. Define radius of curvature. Let $r = f(\theta)$ be a polar form of a curve with a point P on it. Then prove that the radius of curvature at P is given by

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2},$$

where $r_1 = f'(\theta)$ and $r_2 = f''(\theta)$ **6**

OR

Q: 4 [A] Show that the intrinsic equation of the curve $y^3 = ax^2$, is $27s = 8a(\sec^3\psi - 1)$ 3

[B] Prove that the length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ measured from $(0, a)$ to the point (x, y) is given by $\frac{3}{2}(ax^2)^{1/3}$ 3

Q: 5. Define a homogeneous function. Also state and prove the Euler's theorem for functions of three variables. 6

OR

Q: 5 [A] If $z = xyf(\frac{y}{x})$ and z is constant, then show that

$$\frac{f'(\frac{y}{x})}{f(\frac{y}{x})} = \frac{x[y + x\frac{dy}{dx}]}{y[y - x\frac{dy}{dx}]}.$$

3

[B] If $H = f(2x - 3y, 3y - 4z, 4z - 2x)$, then prove that

$$\frac{1}{2}\frac{\partial H}{\partial x} + \frac{1}{3}\frac{\partial H}{\partial y} + \frac{1}{4}\frac{\partial H}{\partial z} = 0.$$

3