V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2014-15

Subject : Mathematics   US02CMTH02   Max. Marks : 25

Matrix Algebra and Differential Equations

Date: 19/03/2015   Timing: 12.30 pm - 1.30 pm

Instructions : (1) This question paper contains FOUR questions

(2) The figures to the right side indicate full marks of the corresponding question/s

(3) The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[1] If $P = \begin{bmatrix} 4 & -1 \\ 3 & 5 \end{bmatrix}$ then $PP' =$

[A] $\begin{bmatrix} 17 & 7 \\ 7 & 34 \end{bmatrix}$ [B] $\begin{bmatrix} -17 & 7 \\ 7 & -34 \end{bmatrix}$ [C] $\begin{bmatrix} 7 & 34 \\ 17 & 7 \end{bmatrix}$ [D] I

[2] Matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 0 \\ 4 & 0 & 1 \end{bmatrix}$ is a

[A] scalar matrix   [B] diagonal matrix

[C] symmetric matrix   [D] skew-symmetric matrix

[3] A square matrix A is said to be an orthogonal matrix if

[A] $AA' = I$ [B] $AA^{\theta} = I$ [C] $AA^{-1} = I$ [D] $A = A'$

Q: 2. Answer any TWO of the following. **4**

[1] Define : (i) Column Matrix (ii) Hermitian Matrix

[2] If A is Hermitian then prove that $B^{\theta}AB$ is Hermitian

[3] Find the characteristic equation of $\begin{bmatrix} 5 & 1 & 1 \\ 2 & 4 & 1 \\ 1 & 4 & 1 \end{bmatrix}$

[4] Find the characteristic roots of $\begin{bmatrix} 5 & 8 \\ 1 & 9 \end{bmatrix}$

Q: 3 [A] Prove that every square matrix can be expressed in one and only one way as $P + iQ$, where P and Q are Hermitian matrices. **5**

[B] If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, then find the values of α and β such that $(\alpha I + \beta A)^2 = A$ **4**

OR

Q: 3 [A] Prove that every square matrix can be expressed in one and only one way as a sum of a symmetric and a skew-symmetric matrix. **5**

[B] State and prove associative law for product of matrices 4

Q: 4 [A] State and prove *Cayley-Hamilton theorem* 5

[B] Find the characteristic roots and any one of the characteristic vectors of :

$$\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

4

OR

Q: 4 [A] Show that the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ satisfies *Cayley-Hamilton* theorem. Hence or otherwise obtain A^{-1}. 5

[B] Prove that every orthogonal matrix A can be expressed as $A = (I + S)(I - S)^{-1}$ by a suitable choice of real skew-symmetric matrix S provided that -1 is not a characteristic root of A 4