

# V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2014-15

Subject : Mathematics

US02CMTH02

Max. Marks : 25

Matrix Algebra and Differential Equations

Date: 19/03/2015

Timing: 12.30 pm - 1.30 pm

- Instructions : (1) This question paper contains FOUR questions  
(2) The figures to the right side indicate full marks of the corresponding question/s  
(3) The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices.

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[ 1 ] If  $P = \begin{bmatrix} 4 & -1 \\ 3 & 5 \end{bmatrix}$  then  $PP' =$

[A]  $\begin{bmatrix} 17 & 7 \\ 7 & 34 \end{bmatrix}$

[B]  $\begin{bmatrix} -17 & 7 \\ 7 & -34 \end{bmatrix}$

[C]  $\begin{bmatrix} 7 & 34 \\ 17 & 7 \end{bmatrix}$

[D] I

[ 2 ] Matrix  $A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 0 \\ 4 & 0 & 1 \end{bmatrix}$  is a

[A] scalar matrix

[B] diagonal matrix

[C] symmetric matrix

[D] skew-symmetric matrix

[ 3 ] A square matrix  $A$  is said to be an orthogonal matrix if

[A]  $AA' = I$

[B]  $AA^{\theta} = I$

[C]  $AA^{-1} = I$

[D]  $A = A'$

Q: 2. Answer any TWO of the following.

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[ 1 ] Define : (i) Column Matrix (ii) Hermitian Matrix

[ 2 ] If  $A$  is Hermitian then prove that  $B^{\theta}AB$  is Hermitian

[ 3 ] Find the characteristic equation of  $\begin{bmatrix} 5 & 1 & 1 \\ 2 & 4 & 1 \\ 1 & 4 & 1 \end{bmatrix}$

[ 4 ] Find the characteristic roots of  $\begin{bmatrix} 5 & 8 \\ 1 & 9 \end{bmatrix}$

Q: 3 [A] Prove that every square matrix can be expressed in one and only one way as  $P + iQ$ , where  $P$  and  $Q$  are Hermitian matrices.

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[ B ] If  $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ , then find the values of  $\alpha$  and  $\beta$  such that  $(\alpha I + \beta A)^2 = A$

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OR

Q: 3 [A] Prove that every square matrix can be expressed in one and only one way as a sum of a symmetric and a skew-symmetric matrix.

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[ B] State and prove associative law for product of matrices

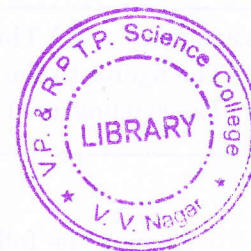
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Q: 4 [A] State and prove *Cayley-Hamilton theorem*

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[ B] Find the characteristic roots and any one of the characteristic vectors of :

$$\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$



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OR

Q: 4 [A] Show that the matrix  $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$  satisfies *Cayley-Hamilton theorem*.

Hence or otherwise obtain  $A^{-1}$ .

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[ B] Prove that every orthogonal matrix  $A$  can be expressed as  $A = (I + S)(I - S)^{-1}$  by a suitable choice of real skew-symmetric matrix  $S$  provided that  $-1$  is not a characteristic root of  $A$

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