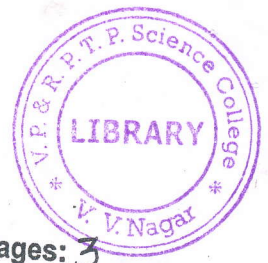


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SEAT No. \_\_\_\_\_



No. of Printed Pages: 3



## Sardar Patel University, Vallabh Vidyanagar

B.Sc. - Semester-VI : Examinations : 2021-22

Subject : Mathematics

US06CMTH23

Max. Marks : 70

Linear Algebra

Date: 06/04/2022, Wednesday

Timing: 03.00 pm - 05.00 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices.

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- [1] Any two generating sets of a vector space must have \_\_\_\_\_.  
 [A] no common elements [B] all different elements [C] same number of elements [D] none
- [2] If a vector space  $V$  with dimension  $n$  contains 3 linearly independent vectors then  $n$  \_\_\_\_\_.  
 [A] = 3 [B] < 3 [C] > 3 [D]  $\geq 3$
- [3] If  $B_1$  and  $B_2$  are two bases for a vector space with dimensions  $m$  and  $n$  then  
 [A]  $m > n$  [B]  $m < n$  [C]  $m = n$  [D]  $m + n = 0$
- [4]  $R^3$  and  $R^2$  are two real vector spaces,  $N(T)$  is null space for a linear transformation  $T : R^3 \rightarrow R^2$  and  $v_1 \neq v_2$  then  $T(v_1) \dots T(v_2)$   
 [A] = [B] < [C] > [D]  $\neq$
- [5] If  $V_1$  and  $V_2$  are two vector spaces with dimensions 2 and 4 then  $L(V_1, V_2) =$  \_\_\_\_\_.  
 [A] 2 [B] 4 [C] 8 [D] 16
- [6] If  $V$  and  $V'$  are vector spaces over fields  $F$  and  $F'$  respectively then a linear transformation  $T : V \rightarrow V'$  can be defined only if \_\_\_\_\_.  
 [A]  $F = F'$  [B]  $F \subset F'$  [C]  $F' \subset F$  [D]  $V = V'$
- [7] Let  $\{v_1, v_2\}$  and  $\{u_1, u_2\}$  be two basis of vector space  $V_2$  respectively. If  $T : V_2 \rightarrow V_2$  is a linear transformation such that  $T(v_1) = 2u_1 + 3u_2$  and  $T(v_2) = u_1 - 2u_2$  then the matrix associated with  $T$  is \_\_\_\_\_  
 [A]  $\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$  [B]  $\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$  [C]  $\begin{bmatrix} -2 & 2 \\ 1 & 3 \end{bmatrix}$  [D]  $\begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$
- [8] Rank of the matrix  $\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$  is \_\_\_\_\_.  
 [A] 0 [B] 1 [C] 2 [D] 3
- [9] If  $A$  is an orthogonal subset of an inner product space then  $u_1 \cdot u_2 - u_2 \cdot u_3 = 0$  for every  $u_1, u_2, u_3 \in A$   
 [A] < [B] > [C]  $\leq$  [D] =
- [10] If  $u, v \in V$ , an inner product space, such that  $u \neq v$  then  $\|u\| \dots \|v\|$   
 [A] < [B] > [C] = [D]  $\neq$

Q: 2. In the following, depending on the type of question either fill in the blank or answer whether a statement is true false

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- [1] If dimension of a vector space is 10 then it cannot contain more than 10 linearly independent vectors. (True/False?)
- [2] A subspace of a vector space is not necessarily an abelian group. (True/False?)
- [3] If  $T$  is a linear transformation on a vector space with dimension 8 and nullity of  $T$  is 2 then its rank is 4. (True/False?)
- [4] If  $T$  is a linear map then for a scalar  $\alpha$  the map  $\alpha^2 T$  is also a linear map. (True/False?)
- [5] Dimensions of real vector spaces  $M_{3 \times 4}$  and  $M_{2 \times 6}$  are equal. (True/False?)
- [6] A matrix associated with a linear transformation  $T : U \rightarrow V$  depends on the basis of vector spaces  $U$  and  $V$  used to represent  $T$ . (True/False?)
- [7] If  $V$  is an inner product space then  $V$  is an abelian group also. (True/False?)
- [8] Every finite dimensional inner product space has an orthogonal basis. (True/False?)

Q: 3. Answer TEN of the following.

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- [1] Define Real Vector Space
- [2] In any vector space prove that  $\alpha u = \bar{0}$  iff either  $\alpha = 0$  or  $u = \bar{0}$ .
- [3] Define (1) Subspace of a vector space. (2) Span.
- [4] Examine whether  $T : R^2 \rightarrow R^3$ ,  $T(x, y) = (x, y, 0)$  is a linear map.
- [5] If  $T : R^3 \rightarrow R^4$  is a one-one linear map then find the nullity of  $T$ .
- [6] Define (1) Nullity of a linear transformation (2) Non-Singular Linear Map.
- [7] Define Matrix Associated with a linear Transformation
- [8] Prove that the columns of a square matrix are LI iff rows of the matrix are LI.
- [9] Find the nullity of  $\begin{bmatrix} 3 & -4 \\ 4 & 2 \end{bmatrix}$
- [10] Define (1) Orthogonal set of vectors (2) Projection of a vector
- [11] Let  $V$  be a real inner product space,  $u, v$  and  $w$  be any three vectors in  $V$  and  $\alpha$  a scalar. Then prove the following.
  - (i)  $(u + v) \cdot w = u \cdot w + v \cdot w$
  - (ii)  $u \cdot (\alpha v) = \alpha(u \cdot v)$
- [12] Define Inner Product Space



Q: 4. Attempt ANY FOUR of the following questions.

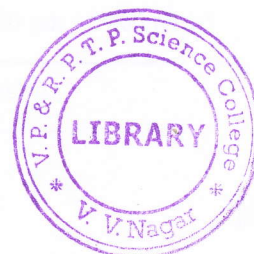
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[1] Prove that a subset  $S$  of a vector space  $V$  is a subspace of  $V$  iff the following conditions are satisfied.

- (a) if  $u, v \in S$  then  $u + v \in S$
- (b) if  $\alpha$  is a scalar and  $u \in V$  then  $\alpha u \in V$

[2] Let  $V$  be a vector space. Prove that,

- (a) The set  $\{v\}$  is L.D. iff  $v = \vec{0}$
- (b) The set  $\{v_1, v_2\}$  is L.D. iff  $v_1, v_2$  are collinear.
- (c) The set  $\{v_1, v_2, v_3\}$  is L.D. iff  $v_1, v_2, v_3$  are coplanar.



[3] Let  $T : U \rightarrow V$  be a linear transformation. Then prove the following

- (1)  $N(T)$  is a subspace of  $U$
- (2)  $R(T)$  is a subspace of  $V$
- (3)  $T$  is one-one iff  $N(T)$  is a zero subspace of  $U$ . i.e.  $N(T) = \{0_U\}$

[4] Let  $T : R^3 \rightarrow R^3$  be a linear transformation defined by  $T(x, y, z) = (x + y + z, y + z, z)$ . Then find  $N(T)$ ,  $R(T)$ ,  $n(T)$  and  $r(T)$ . Also verify rank-nullity theorem.

[5] Let a linear transformation  $T : V_2 \rightarrow V_3$  be defined by

$$T(x_1, x_2) = (x_1 + x_2, 2x_1 - x_2, 7x_2)$$

Find the matrix associate with linear transformation relative to standard bases  $B_1 = \{e_1, e_2\}$  and  $B_2 = \{f_1, f_2, f_3\}$  of  $V_2$  and  $V_3$  respectively.

[6] Let  $T_1, T_2 : U \rightarrow V$  be two linear maps and  $B_1$  and  $B_2$  be their ordered bases respectively. Prove that

$$(\alpha_1 T_1 + \alpha_2 T_2 : B_1, B_2) = \alpha_1 (T_1 : B_1, B_2) + \alpha_2 (T_2 : B_1, B_2)$$

[7] Prove that any orthogonal set of non-zero vectors in an inner product space is linearly independent (LI).

[8] Orthonormalise the set of linearly independent vectors  $\{(1, 0, 1, 1), (-1, 0, -1, 1), (0, -1, 1, 1)\}$  of  $V_4$ .

