[166]

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Sardar Patel University, Vallabh Vidyanagar

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Subject : Mathematics US06CMTH23 Max. Marks : 70

Linear Algebra

Date: 06/04/2022, Wedneday Timing: 03.00 pm - 05.00 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. 10

[1] Any two generating sets of a vector space must have ____.
[A] no common elements [B] all different elements [C] same number of elements [D] none

[2] If a vector space V with dimension n contains 3 linearly independents vectors then n ____.
[A] $= 3$ [B] < 3 [C] > 3 [D] $\geqslant 3$

[3] If B_1 and B_2 are two bases for a vector space with dimensions m and n then
[A] $m > n$ [B] $m < n$ [C] $m = n$ [D] $m + n = 0$

[4] R^3 and R^2 are two real vector spaces, $N(T)$ is null space for a linear transformation $T : R^3 \to R^2$ and $v_1 \neq v_2$ then $T(v_1)$ ___ $T(v_2)$
[A] $=$ [B] $<$ [C] $>$ [D] $\neq$

[5] If V_1 and V_2 are two vector spaces with dimensions 2 and 4 then $L(V_1, V_2) =$ ____.
[A] 2 [B] 4 [C] 8 [D] 16

[6] If V and V' are vector spaces over fields F and F' respectively then a linear transformation $T : V \to V'$ can be defined only if ____.
[A] $F = F'$ [B] $F \subset F'$ [C] $F' \subset F$ [D] $V = V'$

[7] Let $\{v_1, v_2\}$ and $\{u_1, u_2\}$ be two basis of vector space V_2 respectively. If $T : V_2 \to V_2$ is a linear transformation such that $T(v_1) = 2u_1 + 3u_2$ and $T(v_2) = u_1 - 2u_2$ then the matrix associated with T is ________.
[A] $\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$ [B] $\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$ [C] $\begin{bmatrix} -2 & 2 \\ 1 & 3 \end{bmatrix}$ [D] $\begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$

[8] Rank of the matrix $\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$ is ___.
[A] 0 [B] 1 [C] 2 [D] 3

[9] If A is an orthogonal subset of an inner product space then $u_1.u_2 - u_2.u_3$ ___ 0 for every $u_1, u_2, u_3 \in A$
[A] $<$ [B] $>$ [C] $\leqslant$ [D] $=$

[10] If $u, v \in V$, an inner product space, such that $u \neq v$ then $\|u\|$ ___ $\|v\|$
[A] $<$ [B] $>$ [C] $=$ [D] $\neq$

P.T.O.

Q: 2. In the following, depending on the type of question either fill in the blank or answer whether a statement is true false 08

[1] If dimension of a vector space is 10 then it cannot contain more than 10 linearly independent vectors. (True/False?)

[2] A subspace of a vector space is not necessarily an abelian group.(True/False?)

[3] If T is a linear transformation on a vector space with dimension 8 and nullity of T is 2 then its rank is 4. (True/False?)

[4] If T is a linear map then for a scalar α the map $\alpha^2 T$ is also a linear map. (True/False?)

[5] Dimensions of real vector spaces $M_{3\times4}$ and $M_{2\times6}$ are equal. (True/False)

[6] A matrix associated with a linear transformation $T : U \to V$ depends on the basis of vector spaces U and V used to represent T. (True/False)

[7] If V is an inner product space then V is an abelian group also. (True/False?)

[8] Every finite dimensional inner product space has an orthogonal basis. (True/False)?

Q: 3. Answer TEN of the following. 20

[1] Define Real Vector Space

[2] In any vector space prove that $\alpha u = \overline{0}$ iff either $\alpha = 0$ or $u = \overline{0}$.

[3] Define (1) Subspace of a vector space. (2) Span.

[4] Examine whether $T : R^2 \to R^3$, $T(x, y) = (x, y, 0)$ is a linear map.

[5] If $T : R^3 \to R^4$ is a one-one linear map then find the nullity of T.

[6] Define (1) Nullity of a linear trnasformation (2) Non-Singular Linear Map.

[7] Define Matrix Associated with a linear Transformation

[8] Prove that the columns of a square matrix are LI iff rows of the matrix are LI.

[9] Find the nullity of $\begin{bmatrix} 3 & -4 \\ 4 & 2 \end{bmatrix}$

[10] Define (1) Orthogonal set of vectors (2) Projection of a vector

[11] Let V be a real inner product space, u, v and w be any three vectors in V and α a scalar. Then prove the following.
(i) $(u + v).w = u.w + v.w$

(ii) $u.(\alpha v) = \alpha(uv)$

[12] Define Inner Product Space

Q: 4. Attempt ANY FOUR of the following questions.

[1] Prove that a subset S of a vector space V is a subspace of V iff the following conditions are satisfied.
(a) if $u, v \in S$ then $u + v \in S$
(b) if α is a scalar and $u \in V$ then $\alpha u \in V$

[2] Let V be a vector space. Prove that,
(a) The set $\{v\}$ is $L.D.$ iff $v = \bar{0}$
(b) The set $\{v_1, v_2\}$ is $L.D.$ iff v_1, v_2 are collinear.
(c) The set $\{v_1, v_2, v_3\}$ is $L.D.$ iff v_1, v_2, v_3 are coplanar.

[3] Let $T : U \to V$ be a linear transformation. Then prove the following
(1) $N(T)$ is a subspace of U
(2) $R(T)$ is a subspace of V
(3) T is one-one iff $N(T)$ is a zero subspace of U. i.e. $N(T) = \{0_U\}$

[4] Let $T : R^3 \to R^3$ be a linear transformation defined by $T(x, y, z) = (x + y + z, y + z, z)$. Then find $N(T), R(T), n(T)$ and $r(T)$. Also verify rank-nullity theorem.

[5] Let a linear transformation $T : V_2 \to V_3$ be defined by

$$T(x_1, x_2) = (x_1 + x_2, 2x_1 - x_2, 7x_2)$$

Find the matrix associate with linear transformation relative to standard bases $B_1 = \{e_1, e_2\}$ and $B_2 = \{f_1, f_2, f_3\}$ of V_2 and V_3 respectively.

[6] Let $T_1, T_2 : U \to V$ be two linear maps and B_1 and B_2 be their ordered bases respectively. Prove that

$$(\alpha_1 T_1 + \alpha_2 T_2 : B1, B2) = \alpha_1 (T_1 : B1, B2) + \alpha_2 (T_2 : B1, B2)$$

[7] Prove that any orthogonal set of non-zero vectors in an inner product space is linearly independent (LI).

[8] Orthonormalise the set of linearly independent vectors $\{(1, 0, 1, 1), (-1, 0, -1, 1), (0, -1, 1, 1)\}$ of V_4.

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