SEAT No.__________ No. of Printed Pages : 2

[131] **SARDAR PATEL UNIVERSITY**

B.Sc.(SEMESTER-VI) EXAMINATION-2022

April 4, 2022 Monday 3:00 p.m. to 5:00 p.m.

US06CMTH21(Complex Analysis) Maximum Marks: 70

Q.1 Choose the correct option in the following multiple choice questions. [10]

(1) Domain of $f(z) = \frac{1}{z^2+4}$ is ....
(A) $\mathbb{C} - \{\pm 2i\}$ (B) $\mathbb{C} - \{\pm 2\}$ (C) $\mathbb{C} - \{\pm i\}$ (D) $\mathbb{C} - \{\pm 1\}$

(2) In the Cartesian form of $f(z) = z^2 + 1$, $Re\{f(z)\} =$ ....
(A) $x^2+y^2+1-2ixy$ (B) x^2+y^2+1 (C) x^2-y^2+1 (D) x^2-y^2+i2xy

(3) $Im(z + z^{-1}) =$ ....
(A) $\left(y - \frac{y}{x^2+y^2}\right)$ (B) $\left(x - \frac{x}{x^2+y^2}\right)$ (C) $\left(x + \frac{x}{x^2+y^2}\right)$ (D) $\left(y + \frac{y}{x^2+y^2}\right)$

(4) If $C-R$ equations are not satisfied at z_0 then $f(z)$ is .... at z_0.
(A) differentiable (B) not differentiable (C) must continuous (D) none

(5) For complex function $f(z) = v + iu$ where $v = v(x, y)$, $u = u(x, y)$, the $C-R$ equations are ......
(A) $u_x = v_y;\ u_y = -v_x$ (B) $u_y = v_x;\ u_x = -v_y$
(C) $u_x = v_y;\ u_y = v_x$ (D) $u_x = v_x;\ u_y = -v_x$

(6) if e^z is real then $Imz =$ .... , $n \in \mathbb{Z}$.
(A) 2π (B) $n\pi$ (C) π (D) n

(7) $i \sin iy =$ ....
(A) $-\sinh y$ (B) $i \sinh y$ (C) $-i \sinh y$ (D) $\cos iy$

(8) $Im\ (Log(3-4i)) =$ ....
(A) $\ln(25)$ (B) $\ln(5)$ (C) π (D) $-\pi/4$

(9) Image of $x = c_1$, $(c_1 \neq 0)$ under the transformation $w = 1/z$ is circle with centre ....
(A) $(0\ ,\ 1/2c_1)$ (B) $(c_1\ ;\ 0)$ (C) $(1/c_1\ ,\ 0)$ (D) $(1/2c_1\ ,\ 0)$

(10) Image of $x > 0$ under the transformation $w = i/z$ is ....
(A) $u < 0$ (B) $u > 0$ (C) $v < 0$ (D) $v > 0$

Q.2 Do as directed. [08]

(1) The polar form of $z = 1 - \sqrt{3}i$ is = ...
(2) $f(z) = z^2 + 2z - 1$ is differentiable only at $z \in \mathbb{C}$ (True or False).
(3) $u(x, y) = x^2 - y^2$ is harmonic function (True or False).
(4) Singular point of $f(z) = \frac{2z}{z(z^2-1)}$ are $z =$ ....
(5) If $e^z = 2 - i2\sqrt{3}$ then $Re(z) =$ ....

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(6) $\log(i^3) = 3\log(i)$ (True or False).

(7) Fixed point of $w = \frac{6z-9}{z}$ are 3 (True or False).

(8) Image of $y < 0$ under the transformation $w = (1+i)z$ is $v < u$ (True or False).

Q.3 Answer the following in short. (Attempt any 10) [20]

(1) By using definition, prove that $\frac{d}{dz}(z) = 1$.

(2) Explain Continuous complex function with example.

(3) Express $f(z) = x^2 - y^2 - 2y + i(2x - 2xy)$ in the terms of z, where $z = x + iy$.

(4) Define: Singular point & Harmonic function.

(5) Verify $f(z) = z^3$ is entire or not.

(6) Prove that $u = e^x \cos y$ is harmonic function.

(7) Evaluate: $log(-1+\sqrt{3}i)$ and $Log(-1+\sqrt{3}i)$.

(8) Prove that $cosz = cosxcoshy - isinxsinhy$.

(9) Prove that $|cosz|^2 = cos^2x + sinh^2y$.

(10) Define: Linear transformation.

(11) Prove that $w = z + B$, where B is complex constant, gives a translation by means of vectors representing B.

(12) Prove that the general linear transformation $w = Az + B$, $A \neq 0$, A and B are complex constant, gives expansion or contraction and a rotation followed by a translation.

Q.4 Answer the following questions. (Attempt any 4) [32]

(1) If $f(z) = \frac{x^3y(y-ix)}{z(x^6+y^2)}$, $z \neq 0$, $f(0) = 0$ (i) Is $\lim_{z\to 0} f(z)$ exists ? (ii) Is f(z) continuous at 0 ? (iii) Is f(z) differentiable at 0 ?

(2) Prove that $f(z) = |z|^2$ is differentiable only at z=0.Also prove that $f'(0) = 0$.

(3) Prove that $u(x,y)$ is harmonic in some domain and find a harmonic conjugate $v(x,y)$ for $u(x,y)$. Also find corresponding analytic function $f(z)$, where $u(x,y) = x^2 - y^2$.

(4) State and prove sufficient conditions for differentiability of $f(z)$.

(5) Prove that (i) $sin^{-1}z = -ilog[iz + \sqrt{1-z^2}]$ (ii) $Log(-1+i) = \frac{1}{2}ln2 + 3\frac{\pi}{4}i$.

(6) Prove that $e^w = z$ iff w has one of the values $w = lnr + i(\Theta + 2n\pi)$; $n \in \mathbb{Z}$. Also find the value of $\log(-1)$ & $Log1$.

(7) Find linear fractional transformation that maps the points: $z_1 = 2$, $z_2 = i$, $z_3 = -2$ *on to* $w_1 = 1$, $w_2 = i$, $w_3 = -1$.

(8) Prove that the set of all bilinear map forms a non-commutative group with respect to composition of mapping.

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