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SARDAR PATEL UNIVERSITY

BSc Examination [Semester: V]

Subject: Physics Course: US05CPHY22

Mathematical Methods

Date: 24-11-21, Wednesday Time: 03.00 pm to 05.00 pm Total Marks: 70

INSTRUCTIONS:

1 Attempt all questions.
2 The symbols have their usual meaning.
3 Figures to the right indicate full marks.

Q-1 Multiple Choice Questions: [Attempt all] **10**

(i) The curvilinear coordinate system will be orthogonal if the ______ of two perpendicular vectors is zero.

(a) Summation (b) Dot product
(c) Multiplication (d) Cross product

(ii) For curvilinear coordinates $\frac{\partial \vec{r}}{\partial v} \times \frac{\partial \vec{r}}{\partial w} =$ ______.

(a) $\frac{h_2 h_3}{h_1} \frac{\partial \vec{r}}{\partial u}$ (b) $\frac{h_1 h_2}{h_3} \frac{\partial \vec{r}}{\partial u}$
(c) $\frac{h_1}{h_2 h_3} \frac{\partial \vec{r}}{\partial u}$ (d) $\frac{h_3 h_1}{h_2} \frac{\partial \vec{r}}{\partial u}$

(iii) ______ is true for Legendre's equation.

(a) $k = n - 1$ or $k = -n - 1$ (b) $k = n$ or $k = -n$
(c) $k = n + 1$ or $k = -n + 1$ (d) $k = n$ or $k = -n - 1$

(iv) For Hermite's function, $H_0(x) =$ ______.

(a) -2 (b) -1
(c) 0 (d) 1

(v) Eigen value of the vibrating string is ______.

(a) $\lambda_n = \frac{n\pi c}{l}$ (b) $\lambda_n = \frac{n\pi l}{c}$
(c) $\lambda_n = \frac{n\pi}{cl}$ (d) $\lambda_n = \frac{2n\pi}{cl}$

(vi) The coefficients α_n for a Fourier series of a periodic function $f(x)$ in $[-\infty, \infty]$ is ______.

(a) $i(a_n + a_{-n})$ (b) $(a_n + a_{-n})$
(c) $i(a_n - a_{-n})$ (d) $(a_n - a_{-n})$

(vii) Fourier equation of heat flow is ______.

(a) $\frac{\partial u}{\partial t} = h^2 \nabla^2 u$ (b) $\frac{\partial u}{\partial t} = h^2 \nabla u$
(c) $\frac{\partial^2 u}{\partial t^2} = h^2 \nabla^2 u$ (d) $\frac{\partial^2 u}{\partial t^2} = h^2 \nabla u$

(viii) In the Simpson's $\frac{1}{3}^{rd}$ rule, we have to use two subintervals of _____ width.

(a) Equal (b) Opposite
(c) Different (d) None of these

(ix) The forward difference operator Δ defined as _______.

(a) $\Delta y_i = y_i - y_{i-1}$ (b) $\Delta y_i = y_{i-1} - y_i$
(c) $\Delta y_i = y_{i+1} - y_i$ (d) $\Delta y_i = y_i - y_{i+1}$

(x) "The best representative curve to the given set of the observed data or observations is one for which E, the sum of the squares of the residuals, is minimum". This concept is known as the _______.

(a) Interpolation (b) Extrapolation
(c) Principle of least squares (d) Curve fitting

Q-2 State True or False. [Attempt all] **8**

(1) For the spherical polar coordinate system, the unit vectors are $\hat{e}_r, \hat{e}_\theta$ and $\hat{e}_\emptyset$.

(2) For cylindrical coordinates $ds^2 = dr^2 + r^2 d\theta^2 + dz^2$.

(3) $P_n(\mu)$ is the coefficient of $J_n(\mu)$ in the expansion of $(1 - 2\mu h + h^2)^{-1/2}$.

(4) Legendre's differential equation is given by $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$.

(5) The phase angle is given by $\emptyset_n = log\left(\frac{\beta_n}{\alpha_n}\right)$.

(6) The rms or effective value of the function f over a period τ is given by $f_E^2 = \frac{1}{\tau}\int_0^\tau f^2(t)dt$.

(7) The shift operator E is defined as $Ef(x) = f(x + h)$.

(8) $y = ax^2 + bx + c$ be the equation of parabola.

Q-3 Answer the following questions in short. (Attempt any ten) **20**

(1) Write down Laplacian in terms of orthogonal curvilinear coordinates.

(2) If $u = 2x + 1,\ v = 3y - 1,\ w = z + 2$, show that u, v, w are orthogonal.

(3) Write equivalent expressions for gradient and divergence in terms of rectangular coordinates.

(4) Show that: $P_n(-\mu) = (-1)^n P_n(\mu)$.

(5) For Bessel's function, prove that: $xJ_n'(x) = -n J_n(x) + xJ_{n-1}(x)$.

(6) Show that: $2nH_{n-1}(x) = H_n'(x)$

(7) Write one dimensional wave equation.

(8) Write telegraphy equation.

(9) Write sine series for $f(x)$ when $0 \le x \le \pi$. (Note: derivation is not required)

(10) Define interpolation.

(11) Derive an equivalent equation of a straight line for $y = ae^{bx}$.

(12) For a shift operator E, show that $\nabla = \frac{E-1}{E}$.

Q.4 Long Answer Questions. (Attempt any four) **32**

(1) Derive expression of gradient in terms of orthogonal curvilinear system.

(2) Prove that the product of sets of two triads of mutually orthogonal vectors are reciprocal to each other.

(3) Derive the series solution of Legendre differential equation in the form of descending power of x.

(4) State and Derive the Rodrigue's formula.

(5) Write the Fourier series for a periodic function $f(x)$ defined in the interval $[-\pi, \pi]$. Derive the coefficients a_0, a_n and b_n of the series.

(6) Obtain the Fourier series for a function $f(x) = x\sin x$, in the interval $-\pi < x < \pi$.

Deduce that: $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1\cdot3} - \frac{1}{3\cdot5} + \frac{1}{5\cdot7} - \cdots$

(7) Using Lagrange's interpolation formula, evaluate $f(5)$ from the given data.

x	1	3	4	6
$y = f(x)$	-3	0	30	132

(8) Using Simpson's 1/3 rule find the approximate value of $y = \int_0^{\pi} \sin x\, dx$ by dividing the range of integration into six equal parts. What is the analytical value of $y = \int_0^{\pi} \sin x\, dx$.

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