SEAT No.__________ No. of Printed Pages : 03

[163] **SARDAR PATEL UNIVERSITY (B. Sc. Sem.5 Examination)**

MATHEMATICS - US05CMTH24

METRIC SPACES AND TOPOLOGICALSPACES

26th November 2021, Friday

Time: 03:00 to 05:00p.m. **Total Marks: 70**

Note: Figures to the right indicates the full marks.

Q:1 Answer thefollowing by selecting the correct choice from the given options. [10]

1. The set (5,7) is ________
   (a) closed (b) countable (c) uncountable (d) unbounded
2. A set {1,3,6,10, ...} is ______
   (a) countable (b) uncountable (c) bounded (d) finite
3. In metric space (R, d), $B[4, \frac{3}{4}]$=________
   (a) $\emptyset$ (b) {0} (c) $(\frac{13}{4}, \frac{19}{4})$ (d) {4}
4. __________ is u −open
   (a) [2,4] (b) R (c) [5,7) (d) none
5. In a topological space ______ of closed set is closed
   (a) intersection (b) finite union
   (c) finite intersection (d) arbitrary union
6. __________ is u −closed set
   (a) (4,13) (b) (2,4] (c) {7} (d) none of these
7. (X, τ) is a topological space, $A \subset X$ then A is dense in X if _____
   (a) $A = X$ (b) $\bar{A} \subset X$
   (c) $A' = X$ (d) $\bar{A} = X$
8. Int(A) is the ________ open subset of A.
   (a) largest (b) smallest (c) highest (d) none
9. The space (R, u) is ______
   (a) connected (b)disconnected
   (c) homeomorphic (d) bicontinuos
10. (R, u) and $((5,7), u_{(5,7)})$ are ________
    (a) compact (b) disconnected
    (c) homeomorphic (d) homeomorphism

Q:2 Answer the given statement is TRUE or FALSE [08]

1. Infinite subset of a countable set is countable
2. If ρ and σ be two metrics on M then $\rho - \sigma$ is also a metric on M.
3. Half open intervals are neither u −open nor u −closed

4. On a set that contains at least three elements, we can always define at least three trivial topologies.
5. $(X, \tau)$is a topological space. $A \subset X$ then A is τ −open set iff Int$(A) \subset A$
6. $(\mathbb{R}, u)$ and $(\mathbb{R}, D)$ are homeomorphic
7. Any discrete space that has more than one point is disconnected
8. An image of a connected space is connected

Q:3 Answer ANY TEN of the following. [20]

1. Prove that set of all positive integers is countable
2. Define:Metric
3. Show that if $\{x_n\}$ is convergent sequence in R_d then there exists a positive integer N such that $x_N = x_{N+1} = x_{N+2} = \cdots = x$
4. Check whether the set $A = [0,2)$ is u −open or not
5. Let $X = \{1,3,5,7\}$ and $\tau = \{\emptyset, X, \{3\}, \{5\}, \{1,3\}, \{1,5\}\}$ check whether τ is a topology for X.
6. Let $X = \{1,2,3,4,5\}$, $\tau = \{\emptyset, X, \{3\}, \{5\}, \{3,5\}\}$ check whether the set $\{1,2,4\}$ is τ-closed or not.
7. In a topological space (R, u) check whether $\frac{1}{8}$ is an interior point of $[0,1]$
8. Define: closure of a set
9. Define: continuous function
10. Show that $(X, \mathcal{J})$ is connected
11. Define: Hausdroff space
12. In a T_2 −space(X, τ), if $p \in X$ then prove that $\{p\}$ is τ −closed.

Q:4 AnswerANY FOUR of the following. [32]

(1) Let (M_1, ρ_1) and (M_2, ρ_2) be two metric spaces and let $f: M_1 \to M_2$ then show that f is continuous on M_1iff the inverse image ofevery open set is open

(2) Let (M, d) be a metric space and let $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$ then show that d_1 is a metric on M.

(3) In usual notations prove that (R, u) is a topological space.

(4) Show thatany finite set of real numbers is closed in the usual topology of $\mathbb{R}$.

(5) Find the set of all cluster points of $(1,2)$ in u $-$topology of R.

(6) Let (X, τ) be a topological space and A be a subset of X. A' be the set of all cluster points of A. Prove that A is τ $-$closed iff $A' \subset A$

(7) Prove that topological space(X, τ) is disconnected iff$X$ has non-empty proper subset that is both τ $-$open and τ $-$closed.

(8) Let (X, τ) be a topological space and Y be a subset of X. If the subspace (Y, τ_Y) is connected then prove that subspace $(\bar{Y}, \tau_{\bar{Y}})$ is also connected.

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