



SEAT No. _____

No. of Printed Pages : 03

[163] SARDAR PATEL UNIVERSITY (B. Sc. Sem.5 Examination)

MATHEMATICS - US05CMTH24

METRIC SPACES AND TOPOLOGICAL SPACES

26th November 2021, Friday

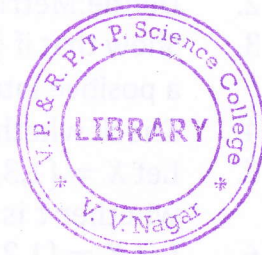
Time: 03:00 to 05:00p.m.

Total Marks: 70

Note: Figures to the right indicates the full marks.

Q:1 Answer the following by selecting the correct choice from [10] the given options.

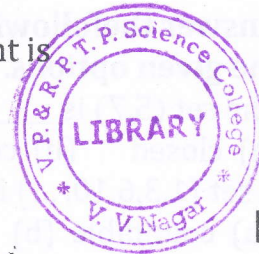
- The set $(5,7)$ is _____
(a) closed (b) countable (c) uncountable (d) unbounded
- A set $\{1,3,6,10, \dots\}$ is _____
(a) countable (b) uncountable (c) bounded (d) finite
- In metric space (R, d) , $B[4, \frac{3}{4}] =$ _____
(a) \emptyset (b) $\{0\}$ (c) $(\frac{13}{4}, \frac{19}{4})$ (d) $\{4\}$
- _____ is u - open
(a) $[2,4]$ (b) R (c) $[5,7)$ (d) none
- In a topological space _____ of closed set is closed
(a) intersection (b) finite union
(c) finite intersection (d) arbitrary union
- _____ is u - closed set
(a) $(4,13)$ (b) $(2,4]$ (c) $\{7\}$ (d) none of these
- (X, τ) is a topological space, $A \subset X$ then A is dense in X if _____
(a) $A = X$ (b) $\bar{A} \subset X$
(c) $A' = X$ (d) $\bar{A} = X$
- $\text{Int}(A)$ is the _____ open subset of A .
(a) largest (b) smallest (c) highest (d) none
- The space (R, u) is _____
(a) connected (b) disconnected
(c) homeomorphic (d) bicontinuos
- (R, u) and $((5,7), u_{(5,7)})$ are _____
(a) compact (b) disconnected
(c) homeomorphic (d) homeomorphism



Q:2 Answer the given statement is TRUE or FALSE [08]

- Infinite subset of a countable set is countable
- If ρ and σ be two metrics on M then $\rho - \sigma$ is also a metric on M .
- Half open intervals are neither u - open nor u - closed

4. On a set that contains at least three elements, we can always define at least three trivial topologies.
5. (X, τ) is a topological space. $A \subset X$ then A is τ -open set iff $\text{Int}(A) \subset A$
6. (\mathbb{R}, u) and (\mathbb{R}, D) are homeomorphic
7. Any discrete space that has more than one point is disconnected
8. An image of a connected space is connected



Q:3 Answer ANY TEN of the following.

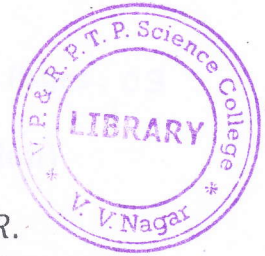
[20]

1. Prove that set of all positive integers is countable
2. Define: Metric
3. Show that if $\{x_n\}$ is convergent sequence in R_d then there exists a positive integer N such that $x_N = x_{N+1} = x_{N+2} = \dots = x$
4. Check whether the set $A = [0, 2)$ is u -open or not
5. Let $X = \{1, 3, 5, 7\}$ and $\tau = \{\emptyset, X, \{3\}, \{5\}, \{1, 3\}, \{1, 5\}\}$ check whether τ is a topology for X .
6. Let $X = \{1, 2, 3, 4, 5\}$, $\tau = \{\emptyset, X, \{3\}, \{5\}, \{3, 5\}\}$ check whether the set $\{1, 2, 4\}$ is τ -closed or not.
7. In a topological space (R, u) check whether $\frac{1}{8}$ is an interior point of $[0, 1]$
8. Define: closure of a set
9. Define: continuous function
10. Show that (X, \mathcal{J}) is connected
11. Define: Hausdroff space
12. In a T_2 -space (X, τ) , if $p \in X$ then prove that $\{p\}$ is τ -closed.

Q:4 Answer ANY FOUR of the following.

[32]

- (1) Let (M_1, ρ_1) and (M_2, ρ_2) be two metric spaces and let $f: M_1 \rightarrow M_2$ then show that f is continuous on M_1 iff the inverse image of every open set is open
- (2) Let (M, d) be a metric space and let $d_1(x, y) = \frac{d(x, y)}{1+d(x, y)}$ then show that d_1 is a metric on M .
- (3) In usual notations prove that (R, u) is a topological space.
- (4) Show that any finite set of real numbers is closed in the usual topology of \mathbb{R} .



- (5) Find the set of all cluster points of $(1,2)$ in u -topology of R .
- (6) Let (X, τ) be a topological space and A be a subset of X . A' be the set of all cluster points of A . Prove that A is τ -closed iff $A' \subset A$
- (7) Prove that topological space (X, τ) is disconnected iff X has non-empty proper subset that is both τ -open and τ -closed.
- (8) Let (X, τ) be a topological space and Y be a subset of X . If the subspace (Y, τ_Y) is connected then prove that subspace $(\bar{Y}, \tau_{\bar{Y}})$ is also connected.

— x —

(3)