SEAT No._____ No. of Printed Pages : 02

[124]

SARDAR PATEL UNIVERSITY (B. Sc. Sem.5 Examination)
MATHEMTICS - US05CMTH21 – REAL ANALYSIS
23rdNovember 2021, Tuesday
Time: 03:00 to 05:00 p.m. Total Marks: 70

Note: Figures to the right indicates the full marks.

Q:1 Answer the following by selecting the correct choice from the given options. **[10]**

1. The smallest member of $\{\frac{1}{n}, n \in N\}$ is _____
   (a) -1 (b) 0 (c) 1 (d) do not exist
2. The lower bound of $\{\frac{(n-1)}{n}, n \in N\}$ is ____
   (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2
3. The set $\{\frac{1}{n}, n \in N\}$ is _____
   (a) open (b) closed (c) neither open nor closed (d) none
4. The interior of the set N =_____
   (a) $\mathbb{N}$ (b) $\mathbb{Z}$ (c) $\mathbb{R}$ (d) $\emptyset$
5. If $\{s_n\} = \{1 + (-1)^n\}, n \in N$ then $\lim_{n\to\infty} \inf s_n$=_____
   (a) 1 (b) 0 (c) -1 (d) does not exist
6. The range set of sequence $\{1 + (-1)^n\}$ is ____
   (a) (0,2) (b) [0,2] (c) [−1,1] (d) {0,2}
7. Every monotonic increasing sequence which is not bounded above _____
   (a) diverges to $-\infty$ (b) diverges to $+\infty$
   (c) converges to $-\infty$ (d) converges to $+\infty$
8. A series with _____ terms is called positive term series
   (a) real (b) negative (c) non-negative (d) none of these
9. A positive term geometric series cannot converge if _____
   (a) $r \leq 1$ (b) $r \leq 1$ (c) $r > 1$ (d) $r \geq 1$
10. The series $\Sigma\frac{1}{n}$ is _____.
    (a) divergent (b) convergent
    (c) converges to 0 (d) none of these

Q:2 Answer the given statement is TRUE or FALSE **[08]**

1. An order structure is always a field structure
2. $\frac{1}{5}$ is the multiplicative inverse of 5 in Z.
3. N is a closed set
4. The set S is always a superset of its interior set

5. Every bounded sequence is convergent
6. A sequence cannot have more than one limit point
7. The series $\Sigma \frac{1}{n^{(n+\frac{1}{n})}}$ is divergent
8. The series $\Sigma \sin\left(\frac{1}{n}\right)$ is divergent

Q:3 Answer ANY TEN of the following. **[20]**

1. Define: Absolute value of a function.
2. State order completeness properties
3. Prove that $|x| < \varepsilon \Leftrightarrow -\varepsilon < x < \varepsilon$
4. Show that interior of S is a subset of S
5. Prove that every open interval is an open set
6. Define: closed set
7. Find the range set of sequence $\{\frac{1}{n}\}$
8. Find limit point of sequence $\{s_n\} = \{1\}$
9. Investigate limit superior of the sequence $\{n^2\}$
10. Test the convergence of the series $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots$
11. Show that a necessary condition for convergence of an infinite series Σu_n is that $\lim_{n\to\infty} u_n = 0$
12. State Cauchy's General principle of convergence.

Q:4 Answer ANY FOUR of the following. **[32]**

(1) Prove that $\sqrt{3}$ is not a rational number
(2) Show that the set of rational numbers is not order completeness.
(3) Prove that the derive set S' of bounded infinite set has the smallest and the greatest member.
(4) Show that every infinite bounded set has a limit point
(5) State and prove Nested Interval theorem.
(6) Prove that sequence $\{r^n\}$ converges iff $-1 < r \leq 1$.
(7) State and prove comparison Test of 2^{nd} type.
(8) Show that the series $\frac{1\cdot2}{3^2\cdot4^2} + \frac{3\cdot4}{5^2\cdot6^2} + \frac{5\cdot6}{7^2\cdot8^2} + \cdots$ is convergent.

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