



[124]

**SARDAR PATEL UNIVERSITY (B. Sc. Sem.5 Examination)**  
**MATHEMATICS - US05CMTH21 - REAL ANALYSIS**

23<sup>rd</sup> November 2021, Tuesday

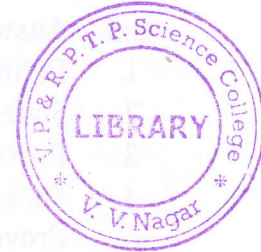
Time: 03:00 to 05:00 p.m.

Total Marks: 70

Note: Figures to the right indicates the full marks.

**Q:1 Answer the following by selecting the correct choice from [10] the given options.**

1. The smallest member of  $\{\frac{1}{n}, n \in \mathbb{N}\}$  is \_\_\_\_\_  
 (a) -1 (b) 0 (c) 1 (d) do not exist
2. The lower bound of  $\{\frac{(n-1)}{n}, n \in \mathbb{N}\}$  is \_\_\_\_\_  
 (a) 0 (b)  $\frac{1}{2}$  (c) 1 (d) 2
3. The set  $\{\frac{1}{n}, n \in \mathbb{N}\}$  is \_\_\_\_\_  
 (a) open (b) closed (c) neither open nor closed (d) none
4. The interior of the set  $N =$  \_\_\_\_\_  
 (a)  $\mathbb{N}$  (b)  $\mathbb{Z}$  (c)  $\mathbb{R}$  (d)  $\emptyset$
5. If  $\{s_n\} = \{1 + (-1)^n\}, n \in \mathbb{N}$  then  $\lim_{n \rightarrow \infty} \inf s_n =$  \_\_\_\_\_  
 (a) 1 (b) 0 (c) -1 (d) does not exist
6. The range set of sequence  $\{1 + (-1)^n\}$  is \_\_\_\_\_  
 (a) (0,2) (b) [0,2] (c) [-1,1] (d) {0,2}
7. Every monotonic increasing sequence which is not bounded above \_\_\_\_\_  
 (a) diverges to  $-\infty$  (b) diverges to  $+\infty$   
 (c) converges to  $-\infty$  (d) converges to  $+\infty$
8. A series with \_\_\_\_\_ terms is called positive term series  
 (a) real (b) negative (c) non-negative (d) none of these
9. A positive term geometric series cannot converge if \_\_\_\_\_  
 (a)  $r < 1$  (b)  $r \leq 1$  (c)  $r > 1$  (d)  $r \geq 1$
10. The series  $\sum \frac{1}{n}$  is \_\_\_\_\_  
 (a) divergent (b) convergent  
 (c) converges to 0 (d) none of these



**Q:2 Answer the given statement is TRUE or FALSE**

**[08]**

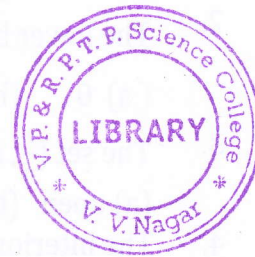
1. An order structure is always a field structure
2.  $\frac{1}{5}$  is the multiplicative inverse of 5 in  $\mathbb{Z}$ .
3.  $\mathbb{N}$  is a closed set
4. The set  $S$  is always a superset of its interior set

5. Every bounded sequence is convergent
6. A sequence cannot have more than one limit point
7. The series  $\sum \frac{1}{n^{(n+\frac{1}{n})}}$  is divergent
8. The series  $\sum \sin\left(\frac{1}{n}\right)$  is divergent

**Q:3 Answer ANY TEN of the following.**

**[20]**

1. Define: Absolute value of a function.
2. State order completeness properties
3. Prove that  $|x| < \varepsilon \Leftrightarrow -\varepsilon < x < \varepsilon$
4. Show that interior of  $S$  is a subset of  $S$
5. Prove that every open interval is an open set
6. Define: closed set
7. Find the range set of sequence  $\left\{\frac{1}{n}\right\}$
8. Find limit point of sequence  $\{s_n\} = \{1\}$
9. Investigate limit superior of the sequence  $\{n^2\}$
10. Test the convergence of the series  $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots$
11. Show that a necessary condition for convergence of an infinite series  $\sum u_n$  is that  $\lim_{n \rightarrow \infty} u_n = 0$
12. State Cauchy's General principle of convergence.



**Q:4 Answer ANY FOUR of the following.**

**[32]**

- (1) Prove that  $\sqrt{3}$  is not a rational number
- (2) Show that the set of rational numbers is not order completeness.
- (3) Prove that the derived set  $S'$  of bounded infinite set has the smallest and the greatest member.
- (4) Show that every infinite bounded set has a limit point
- (5) State and prove Nested Interval theorem.
- (6) Prove that sequence  $\{r^n\}$  converges iff  $-1 < r \leq 1$ .
- (7) State and prove comparison Test of 2<sup>nd</sup> type.
- (8) Show that the series  $\frac{1 \cdot 2}{3^2 \cdot 4^2} + \frac{3 \cdot 4}{5^2 \cdot 6^2} + \frac{5 \cdot 6}{7^2 \cdot 8^2} + \dots$  is convergent.

— X —

(2)