[51] **SEAT No.**________ **No. of Printed Pages : 3**

Sardar Patel University, Vallabh Vidyanagar

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Subject : Mathematics US06CMTH24 Max. Marks : 70

Riemann Integration and Series Of Functions

Date: 19/07/2021, Monday Timing: 10.00 am - 12.00 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. **10**

[1] If f is bounded on $[a, b]$ and $P*$ is a refinement of a partition P of $[a, b]$ then $L(P, f)$ ____ $L(P*, f)$

[A] $>$ [B] $\geqslant$ [C] $<$ [D] $\leqslant$

[2] If P_1 and P_2 are partitions of $[a, b]$ then P_1 is said to be a refinement of P_2 if ____

[A] $P_1 \neq P_2$ [B] $P_1 \subset P_2$ [C] $P_2 \subset P_1$ [D] none

[3] Any two partitions of a closed interval have _____ elements in common

[A] atleast two [B] exactly two [C] all the elements in common [D] no

[4] Every ______ function is integrable.

[A] bounded [B] unbounded [C] discontinuous [D] continuous

[5] If a function f has a finite number limit points of the set of points of discontinuity over $[a, b]$ then

[A] it is monotonic over $[a, b]$ [B] it is not integrable over $[a, b]$

[C] it is integrable over $[a, b]$ [D] none

[6] A continuous function over a closed interval $[a, b]$ is always

[A] an increasing function [B] a decreasing function

[C] a constant function [D] an integrable function

[7] If $\int_0^{\frac{\pi}{2}} \log \sin x dx =$ _____.

[A] $\frac{\pi}{2}$ [B] $-\frac{\pi}{2}$ [C] $\frac{\pi}{2} \log 2$ [D] $-\frac{\pi}{2} \log 2$

[8] If $\int_{-1}^{3} \frac{1}{(x+1)^n} dx$ converges iff ____.

[A] $n \leqslant 1$ [B] $n \geqslant 1$ [C] $n > 1$ [D] $n < 1$

[9] $\{nx\}$ converges pointwise to _____ for $x \in (1, 2)$.

[A] 0 [B] 1 [C] 2 [D] none

[10] $\sum \frac{x}{n^{k-2}}$ converges uniformly for $k =$ _____ on $x \in [0, 2]$.

[A] 0 [B] 1 [C] 3 [D] 5

Q: 2. In the following, depending on the type of question either fill in the blank or answer whether a statement is true false 08

[1] Any two partitions of a closed interval have atleast two elements in common (True/False?)

[2] If $f(x) = 7$ then $\int_0^5 f.dx =$ ____

[3] For a function to be integrable it is not necessary that it is continuous. (True/False?)

[4] Function $f(x) = \sin x + x^2$ is integrable on $[-1, 0]$. (True/False?)

[5] $\int_0^2 \frac{1}{x(x-2)}dx$ has an infinite discontinuity at 0 and 2 both (True/False?).

[6] $\int_{-1}^1 \frac{1}{x^2+x}dx$ has no infinite discontinuity in $[-1, 1]$ (True/False?).

[7] The sequence of functions $\left\{\frac{x}{2^n}\right\}$ is pointwise convergent on $[0, 1]$. (True/False?)

[8] The sequence of functions $\{x^2\}$ is pointwise convergent on $[2, 3]$. (True/False?)

Q: 3. Answer TEN of the following. 20

[1] Write any two refinements of a partition $\{1, 1.2, 1.3, 1.4, 1.5, 2\}$ of $[1, 2]$

[2] Can two partitions of $[a, b]$ be disjoint? Justify.

[3] Find the mesh of the partition $\{2, 3, 5, 7, 10, 11, 13\}$ of $[2, 13]$

[4] Is $f(x) = [x]$ an integrable function over $[0, 5]$? Justify.

[5] Is $f(x) = x$ an integrable function over $[0, 1]$? Justify.

[6] A function f has infinite number of points of discontinuity but the set of discontinuties has only one limit point in $[2, 8]$. Can it be integrable over $[2, 8]$? Justify.

[7] Define : Improper Integral

[8] Is $\int_0^1 \frac{\sin x}{x}dx$ an improper integral? Justify.

[9] Find the points of infinite discontinuities of $\int_0^5 \frac{1}{x^2-5x+6}dx$.

[10] Define : Pointwise Convergence of a sequence of functions

[11] Define : Uniform convergence of series of functions.

[12] Show that the limit of differentials is not equal to the differential of the limit.

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Q: 4. Attempt ANY FOUR of the following questions.

[1] Show that x^2 is integrable on any interval $[0, k]$

[2] State and prove Darboux's Theorem.

[3] If f_1, f_2 are integrable on $[a, b]$ and c_1 and c_2 any two constants, then prove that $c_1 f_1 + c_2 f_2$ is integrable and

$$\int_a^b (c_1 f_1 + c_2 f_2).dx = \int_a^b c_1 f_1.dx + \int_a^b c_2 f_2.dx$$

[4] If a function f is monotonic on $[a, b]$, then prove that f is integrable on $[a, b]$.

[5] State and prove the comparision test-I for convergence of an improper integral.

[6] Prove that $\int_a^b \frac{1}{(x-a)^n} dx$ converges iff $n < 1$

[7] State and prove Cauchy's criteria for uniform convergence of a sequence of functions.

[8] Test for uniform convergence of the sequence $\{f_n\}$, where $f_n(x) = \dfrac{nx}{1 + n^2 x^2}$

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