SEAT No.________

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[47]

SARDAR PATEL UNIVERSITY

B.Sc. SEM-VI EXAMINATION

17^{th} July 2021 , Saturday

10:00 am to 12:00 noon

Sub: Mathematics (US06CMTH23) (Linear Algebra)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) If $T : V_1 \to V_2$ defined by $T(x) = (x, 0)$ then $T(x + y) =$ ....

(a) (x, y) (b) $(x + y, 0)$ (c) $(x, 0)$ (d) $(y, 0)$

(2) dim $P_3 =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

(3) If a linear map $T : V_2 \to V_2$ defined by $T(x, y) = (x, -y)$, $B_1 \& B_2$ are standard basis for V_2 then $(T : B_1, B_2) =$ ....

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$; (c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(4) Every subset of .... set is LI .

(a) LD (b) LI (c) zero (d) power

(5) In vector space V , $\{v\}$ is LD iff ....

(a) $v = 0$ (b) $v \neq 0$ (c) $v = \{0\}$ (d) $v = 1$

(6) dim $V_3 =$ ....

(a) 1 (b) 2 (c) 3 (d) 0

(7) In any vector space V , $0\,\bar{u} =$ ....

(a) 0 (b) u (c) $\bar{0}$ (d) 1

(8) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $B_1 = B_2 = \{e_1, e_2, e_3\}$,then the linear map T such that $A = (T : B_1, B_2)$ is given by T(x , y , z) = ...

(a) (x , 0 , 0) (b) (0 , y , 0) (c) (0 , 0 , z) (d) (x , y , z)

(9) $T : V_3 \to V_4$ then the matrix $(T : B_1, B_2)$ is of order ....

(a) 4×3 (b) 3×3 (c) 3×4 (d) 4×4

(10) If $T : U \to V$ is linear map then $T(0) =$ ....

(a) 0 (b) 1 (c) 1 (d) 0

Q.2 Do as directed: [08]

(1) True or False: In any vector space V , $\alpha\,\bar{0} = 0$.

(2) True or False: Any set containing zero vector is LD set

(P.T.O.)

(3) True or False: The columns of a square matrix are LI if its rows are LI.

(4) True or False: If $T : V_1 \to V_3$ defined by $T(x) = (x, 2x, 3x)$ then T is a Linear map.

(5) If $T : P \to P$ defined by $T(p) = p'$ then $T(p+q)$............. $T(p) + T(q)$.

(6) Any orthogonal set of non-zero vectors in an inner product space is...... .

(7) The zero is a characteristic root of if and only if is ....

(8) If $T : U \to V$ is linear map then $T(\alpha u_1 + \beta u_2)$..................$\alpha T(u_1) + \beta T(u_2)$, for all scalar α, β, for all $u_1, u_2 \in U$

Q.3 Answer the following in short.[Attempt any **ten**]: [20]

(1) Define Inner product on vector space V.

(2) For any vector space V, Prove that $(-1)u = -u$, forall $u \in V$.

(3) Is $\{(1,2,1), (-1,1,0), (5,-1,2)\}$ LI of V_3 ?

(4) Define : Matrix of linear map T related to the ordered bases B_1 and B_2.

(5) Define an Isomorphism of Linear Transformation.

(6) Is a set$\{p \in P$ / degree of p = 4 $\}$ Subspace of a vector space P ?

(7) Define : Basis for a Vector space.

(8) Define : Linear map associated with the matrix relative to the ordered bases B_1 and B_2.

(9) Is a set $\{(x_1, x_2, x_3) \in V_3 / x_1^2 + x_2^2 + x_3^2 \leq 1\}$ Subspace of a vector space V_3 ?

(10) Is $\{(1,0,0), (2,0,0), (0,0,1)\}$ LI of V_3 ?

(11) Let $T : U \to V$ be a linear map, then prove that $T(0_U) = 0_V$.

(12) Is a map $T : V_3 \to V_2$ defined by $T(x_1, x_2, x_3) = (|x_1|, 0)$ Linear ? Justify.

Q.4 Attempt any Four. [32]

(a) Show that $V_3 = \{(x_1, x_2, x_3) \ / \ x_1, x_2, x_3 \in \mathbb{R}\}$ is a real vector space under usual addition and scalar multiplication.

(b) Show that a linear transformation $T : V \to V'$ is one one mapping iff $Ker(T) = 0$.

(c) Let V be a vector space V with basis consist of n elements then Prove that any $n+1$ elements of V are linerely dependent.

(d) Let $V = P_n(x)$, the set of all polynomials of degree less than or equal to n and let $V' = F$. Is the mapping defined by $T : V \to V'$, where $T(a_o + a_1x + a_2x + \cdots + a_nx) = a_o$ is a linear transformation or not? Justify your answer in detail.

(e) Prove that the set $M_n(F)$ of all $n \times n$ over F forms a ring.

(f) Determine the Eigen values and the corresponding Eigen vectors for the matrix

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

(g) Let $A = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & -1 \\ -2 & 1 & 0 \\ 1 & 2 & -2 \end{bmatrix}$. Obtain a linear map associated with the given matrix , where B_1 and B_2 are standard bases of V_3 and V_4 respectively.

(h) Orthonormalize the set of linearly independent vectors $\{(1,0,1,1),(-1,0,-1,1),(0,-1,1,1)\}$ of V_4.

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