SEAT No.__________ No. of Printed Pages : 03

[46]

SARDAR PATEL UNIVERSITY
BSc Sem-VI Examination
Mathematics
US06CMTH22-Ring Theory

Date: 16-07-2021 Time : 10:00 TO 12:00

Q. 1 Answer the following by selecting correct choice from the options. (10)

1. _____is a skew field but not field.

   A. $\mathbb{Z}$
   B. $\mathbb{Q}$
   C. $M_2(\mathbb{R})$
   D. Ring of Quaternian.

2. ________is a not an integral domain.

   A. $\mathbb{Z}$
   B. $\mathbb{Q}$
   C. $M_2(\mathbb{R})$
   D. All of these

3. ________is a non-commutative ring with unit element.

   A. $\mathbb{Z}$
   B. $\mathbb{Q}$
   C. $M_2(\mathbb{R})$
   D. All of these

4. In $\mathbb{Z}+i\mathbb{Z}$, GCD of 2 and $-1+5i$ is ________

   A. $2+i$
   B. $2-i$
   C. i
   D. $1-i$

5. Quotient field of ring of Gaussian integers is ______.

   A. $\mathbb{Z}+i\mathbb{Z}$
   B. $\mathbb{Q}+i\mathbb{Q}$
   C. $\mathbb{Q}$
   D. $M_2(\mathbb{R})$

6. Every integral domain can be imbedded in a ______

   A. $\mathbb{Z}$
   B. $\mathbb{N}$
   C. *field*
   D. *ring*

(P.T.O.)

7. If F is a field, $f(x) \in F[x], \alpha \in F$ is a root of $f(x)$ then ______

A. $(x-\alpha)|f(x)$

B. $(x+\alpha)|f(x)$

C. $f(x)|(x-\alpha)$

D. $f(x)|(x+\alpha)$

8. If I is an ideal in ring R then unit element in R/I is ____

A. 0

B. 1

C. R

D. $1+I$

9. Let R be an Euclidean domain, $a, b \in R$, a is proper divisor of b then $d(b)$ __ $d(a)$.

A. $=$

B. $\leq$

C. $>$

D. $<$

10. If $R = \mathbb{Z} + i\mathbb{Z}$, $f(x) = 2x^2 - (1+i)x - 2$ then content of f is ____

A. $1+i$

B. $1-i$

C. $2-i$

D. $2+i$

Q.2 State whether the following statements are True or False. **(8)**

1) $\mathbb{Q}$ is not a field.
2) Every ideal is a subring in a ring.
3) The polynomial $1 + x + x^2 \in \mathbb{Q}$ is reducible.
4) Field has proper ideals.
5) $1+i$ is irreducible in $\mathbb{Z} + i\mathbb{Z}$.
6) The polynomial of degree n has exactly $n+1$ roots.
7) If $f(x) = 3x^3 - 2x + 1$ and $g(x) = x^2 - 1$ are polynomials in $\mathbb{R}[x]$ then $deg(fg)$ is 5.
8) Every Euclidean Domain is factorization domain.

Q. 3 Answer any TEN. **(20)**

1) Define zero divisor in a ring.
2) Define Characteristic of a ring.
3) Find Characteristic of $\mathbb{Z}_7$.

4) Define an isomorphism for rings.
5) Define ideal in a ring.
6) Find $\mathbb{Z}/5\mathbb{Z}$.
7) Find the maximal ideal of a field.
8) Check whether $3+\sqrt{-5}$ is irreducible element or not.
9) Define principal ideal.
10) Find $C(f)$ for $f(x)=3x^3-2x^2+6x+9$ in $\mathbb{Z}[x]$.
11) Prove that $x^n-5\in\mathbb{Z}[x]$ is irreducible.
12) Find all roots of x^3+5x in $\mathbb{Z}_6$.

Q.4 Attempt any FOUR. **(32)**

1) Let R be a ring then prove that, $for\ all\ a,b,c\in R$
   $(i)\ a0=0$ $(ii)\ a(-b)=(-a)b=-(ab)$ $(iii) a(b-c)=ab-ac$
2) Let f be a ring homomorphism, then prove that f is one-one if and only if $Kerf=\{0\}$.
3) If R isa commutative ring with 1, then prove that every maximal ideal in R is a prime ideal.
4) State and prove the first isomorphism theorem for ring.
5) Prove that any two elements of U.F.D. have a GCD.
6) Show that the ring of Gaussian integers is an Euclidean domain.
7) Let R be a unique factorization domain, Prove that product of two primitive polynomials over R is also a primitive polynomial.
8) State and prove Eisenstein's criterion.

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