SEAT No.________ No. of Printed Pages : 02

[55]

SARDAR PATEL UNIVERSITY

B.Sc.(SEMESTER-VI) EXAMINATION-2021

July 15, 2021, Thursday

10:00 a.m. to 12:00 p.m.

US06CMTH21(Complex Analysis)

Maximum Marks: 70

Q.1 Choose the correct option in the following multiple choice questions. [10]

(1) Domain of $f(z) = \frac{1}{z^2-1}$ is ....
(A) $\mathbb{C} - \{i\}$ (B) $\mathbb{C} - \{-i\}$ (C) $\mathbb{C} - \{\pm i\}$ (D) $\mathbb{C} - \{\pm 1\}$

(2) Cartesian form of $f(z) = z^2$ is ....
(A) $x^2 + y^2 + 2ixy$ (B) $x^2 + y^2 - 2ixy$ (C) $x^2 - y^2 - i2xy$ (D) $x^2 - y^2 + i2xy$

(3) $f(z) = |z|^2$ is differentiable only at ....
(A) $z = 1$ (B) $z \neq 1$ (C) $z \neq 0$ (D) $z = 0$

(4) If f differentiable at z_0 then C-R equation satisfied at ....
(A) $z = 1$ (B) $z = z_0$ (C) $z = 0$ (D) None of these

(5) Singular points of $f(z) = \frac{z^3+i}{z^2-3z+2}$ are $z =$ ....
(A) 1, 2 (B) $1, i$ (C) 0, 1 (D) $1, 3, i$

(6) $i \sin iy =$ ....
(A) $-\sinh y$ (B) $i \sinh y$ (C) $-i \sinh y$ (D) $\cos iy$

(7) $f(z) = e^{-z}$, $u_x + iv_x =$ ....
(A) e^z (B) e^{-z} (C) $-e^{-z}$ (D) 0

(8) $\lim_{z \to -\infty} expz =$ ....
(A) ∞ (B) 1 (C) 0 (D) -1

(9) Image of $y > 0$ under the transformation $w = i/z$ is ....
(A) $u < 0$ (B) $u > 0$ (C) $v < 0$ (D) $v > 0$

(10) If $T(z) = \dfrac{az+b}{cz+d}$, $(ad - bc \neq 0)$. Then $\lim_{z \to -d/c} T(z) =$ ...., $if\ c \neq 0$.
(A) ∞ (B) i (C) 2 (D) 0

Q.2 Do as directed. [08]

(1) Domain of $f(z) = \dfrac{1}{z^2+1}$ is ....

(2) True or False?
$f(z) = |z|$ is differentiable only at $z = 0$.

(3) True or False?
If $u(x, y) = 2x - x^3 + 3xy^2$ then $u_{xx} + u_{yy} = 1$.

(4) Singular point of $f(z) = \dfrac{2z}{z(z^2+1)}$ are $z =$ ....

(5) $exp(2 \pm 3\pi i) =$ ....

(6) True or False?
$\lim_{z \to \infty} exp(-z) = 0$.

(7) Fixed point of $w = \dfrac{6z-9}{z}$ are ....

(8) True or False?
The image of line $x = c_1$, $c_1 \neq 0$ under the transformation $w = 1/z$ is square.

(P.T.O.)

Q.3 Answer the following in short. (Attempt any 10) [20]

(1) By using definition, prove that $\frac{d}{dz}(c) = 0$,where c is complex constant.
(2) Define: Continuous complex function.
(3) Express $f(z) = x^2 - y^2 - 2y + i(2x - 2xy)$ in the terms of z, where $z = x + iy$
(4) Define: Analytic function & Entire function.
(5) Verify $f(z) = (3x + y) + i(3y - x)$ is entire or not?
(6) Prove that $u = e^x \sin y$ is harmonic function.
(7) Solve: $e^{2z-1} = 1$.
(8) Prove that $\sin z = \sin x \cosh y + i \cos x \sinh y$.
(9) Prove that $\sinh z = \sinh x \cos y + i \cosh x \sin y$.
(10) Define: Linear fractional transformation.
(11) Prove that $w = z + B$, where B is complex constant, gives a translation by means of vectors representing B.
(12) Find the image of line $x \geq c_1$, $c_1 > 0$ under the transformation $w = 1/z$ is the circle. Also show the region graphically.

Q.4 Answer the following questions. (Attempt any 4) [32]

(1) Let $f(z) = u(x, y) + iv(x, y)$, $z_0 = x_0 + iy_0$ and $w_0 = u_0 + v_0$ then prove that $\lim_{z \to z_0} f(z) = w_0$ if and only if $\lim_{(x,y) \to (x_0,y_0)} u(x, y) = u_0$ & $\lim_{(x,y) \to (x_0,y_0)} v(x, y) = v_0$
(2) Give an example of function such that its real and imaginary component have continuous partial derivative of all order at a point but the function is not differentiable at that point. Verify it.
(3) Let $f(z) = u(x, y) + iv(x, y)$ and $f'(z)$ exist at $z_0 = x_0 + iy_0$. Prove that the first order partial derivatives of u and v must exist at (x_0, y_0) and they satisfies the Cauchy-Reimann equations $u_x = v_y$ & $u_y = -v_x$ at (x_0, y_0). Also prove that $f'(z) = u_x + iv_x$ where u_x and v_x are evaluated at (x_0, y_0).
(4) State and prove sufficient conditions for differentiability of $f(z)$.
(5) Prove that (i) $cosh^{-1}z = log[z + \sqrt{z^2 - 1}]$ (ii) $tanh^{-1}z = \frac{1}{2} log\left[\frac{1+z}{1-z}\right]$.
(6) Prove that $e^w = z$ iff w has one of the values $w = lnr + i(\Theta + 2n\pi)$; $n \in \mathbb{Z}$. Also find the value of $log1$ & $Log1$.
(7) Find linear fractional transformation that maps the points: $z_1 = 2$, $z_2 = i$, $z_3 = -2$ onto $w_1 = 1$, $w_2 = i$, $w_3 = -1$.
(8) Prove that the transformation $w = \sin z$ is a one-one mapping of the semi infinite strip $y \geq 0$. $-\pi/2 \leq c_1 \leq \pi/2$ in the z-plane onto the upper half $v \geq 0$ of the w-plane.

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