

SEAT No. \_\_\_\_\_

No. of Printed Pages : 02

[55]

**SARDAR PATEL UNIVERSITY**

B.Sc.(SEMESTER-VI) EXAMINATION-2021

July 15, 2021, Thursday

10:00 a.m. to 12:00 p.m.

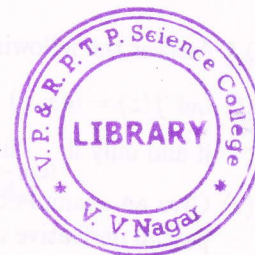
US06CMTH21(Complex Analysis)

Maximum Marks: 70

Q.1 Choose the correct option in the following multiple choice questions.

[10]

- (1) Domain of  $f(z) = \frac{1}{z^2-1}$  is .....  
(A)  $\mathbb{C} - \{i\}$  (B)  $\mathbb{C} - \{-i\}$  (C)  $\mathbb{C} - \{\pm i\}$  (D)  $\mathbb{C} - \{\pm 1\}$
- (2) Cartesian form of  $f(z) = z^2$  is .....  
(A)  $x^2 + y^2 + 2ixy$  (B)  $x^2 + y^2 - 2ixy$  (C)  $x^2 - y^2 - i2xy$  (D)  $x^2 - y^2 + i2xy$
- (3)  $f(z) = |z|^2$  is differentiable only at .....  
(A)  $z = 1$  (B)  $z \neq 1$  (C)  $z \neq 0$  (D)  $z = 0$
- (4) If  $f$  differentiable at  $z_0$  then C-R equation satisfied at .....  
(A)  $z = 1$  (B)  $z = z_0$  (C)  $z = 0$  (D) None of these
- (5) Singular points of  $f(z) = \frac{z^3+i}{z^2-3z+2}$  are  $z =$  .....  
(A) 1, 2 (B) 1,  $i$  (C) 0, 1 (D) 1, 3,  $i$
- (6)  $i \sin iy =$  .....  
(A)  $-\sinh y$  (B)  $i \sinh y$  (C)  $-i \sinh y$  (D)  $\cos iy$
- (7)  $f(z) = e^{-z}$ ,  $u_x + iv_x =$  .....  
(A)  $e^z$  (B)  $e^{-z}$  (C)  $-e^{-z}$  (D) 0
- (8)  $\lim_{z \rightarrow -\infty} \exp z =$  .....  
(A)  $\infty$  (B) 1 (C) 0 (D) -1
- (9) Image of  $y > 0$  under the transformation  $w = i/z$  is .....  
(A)  $u < 0$  (B)  $u > 0$  (C)  $v < 0$  (D)  $v > 0$
- (10) If  $T(z) = \frac{az+b}{cz+d}$ , ( $ad-bc \neq 0$ ). Then  $\lim_{z \rightarrow -d/c} T(z) =$  ....., if  $c \neq 0$ .  
(A)  $\infty$  (B)  $i$  (C) 2 (D) 0



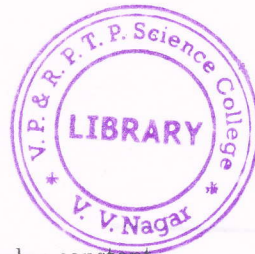
Q.2 Do as directed.

[08]

- (1) Domain of  $f(z) = \frac{1}{z^2+1}$  is .....
- (2) True or False?  
 $f(z) = |z|$  is differentiable only at  $z = 0$ .
- (3) True or False?  
If  $u(x, y) = 2x - x^3 + 3xy^2$  then  $u_{xx} + u_{yy} = 1$ .
- (4) Singular point of  $f(z) = \frac{2z}{z(z^2+1)}$  are  $z =$  .....
- (5)  $\exp(2 \pm 3\pi i) =$  .....
- (6) True or False?  
 $\lim_{z \rightarrow \infty} \exp(-z) = 0$ .
- (7) Fixed point of  $w = \frac{6z-9}{z}$  are .....
- (8) True or False?  
The image of line  $x = c_1$ ,  $c_1 \neq 0$  under the transformation  $w = 1/z$  is square.

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(P.T.O.)



Q.3 Answer the following in short. (Attempt any 10)

[20]

- (1) By using definition, prove that  $\frac{d}{dz}(c) = 0$ , where  $c$  is complex constant.
- (2) Define: Continuous complex function.
- (3) Express  $f(z) = x^2 - y^2 - 2y + i(2x - 2xy)$  in the terms of  $z$ , where  $z = x + iy$
- (4) Define: Analytic function & Entire function.
- (5) Verify  $f(z) = (3x + y) + i(3y - x)$  is entire or not?
- (6) Prove that  $u = e^x \sin y$  is harmonic function.
- (7) Solve:  $e^{2z-1} = 1$ .
- (8) Prove that  $\sin z = \sin x \cosh y + i \cos x \sinh y$ .
- (9) Prove that  $\sinh z = \sinh x \cos y + i \cosh x \sin y$ .
- (10) Define: Linear fractional transformation.
- (11) Prove that  $w = z + B$ , where  $B$  is complex constant, gives a translation by means of vectors representing  $B$ .
- (12) Find the image of line  $x \geq c_1$ ,  $c_1 > 0$  under the transformation  $w = 1/z$  is the circle. Also show the region graphically.

Q.4 Answer the following questions. (Attempt any 4)

[32]

- (1) Let  $f(z) = u(x, y) + iv(x, y)$ ,  $z_0 = x_0 + iy_0$  and  $w_0 = u_0 + v_0$  then prove that  $\lim_{z \rightarrow z_0} f(z) = w_0$  if and only if  $\lim_{(x,y) \rightarrow (x_0,y_0)} u(x, y) = u_0$  &  $\lim_{(x,y) \rightarrow (x_0,y_0)} v(x, y) = v_0$
- (2) Give an example of function such that its real and imaginary component have continuous partial derivative of all order at a point but the function is not differentiable at that point. Verify it.
- (3) Let  $f(z) = u(x, y) + iv(x, y)$  and  $f'(z)$  exist at  $z_0 = x_0 + iy_0$ . Prove that the first order partial derivatives of  $u$  and  $v$  must exist at  $(x_0, y_0)$  and they satisfies the Cauchy-Reimann equations  $u_x = v_y$  &  $u_y = -v_x$  at  $(x_0, y_0)$ . Also prove that  $f'(z) = u_x + iv_x$  where  $u_x$  and  $v_x$  are evaluated at  $(x_0, y_0)$ .
- (4) State and prove sufficient conditions for differentiability of  $f(z)$ .
- (5) Prove that (i)  $\cosh^{-1} z = \log[z + \sqrt{z^2 - 1}]$  (ii)  $\tanh^{-1} z = \frac{1}{2} \log \left[ \frac{1+z}{1-z} \right]$ .
- (6) Prove that  $e^w = z$  iff  $w$  has one of the values  $w = \ln z + i(\Theta + 2n\pi)$ ;  $n \in \mathbb{Z}$ . Also find the value of  $\log 1$  &  $\text{Log} 1$ .
- (7) Find linear fractional transformation that maps the points:  
 $z_1 = 2$ ,  $z_2 = i$ ,  $z_3 = -2$  onto  $w_1 = 1$ ,  $w_2 = i$ ,  $w_3 = -1$ .
- (8) Prove that the transformation  $w = \sin z$  is a one-one mapping of the semi infinite strip  $y \geq 0$ ,  $-\pi/2 \leq x \leq \pi/2$  in the  $z$ -plane onto the upper half  $v \geq 0$  of the  $w$ -plane.

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