

SEAT No. \_\_\_\_\_

No. of Printed Pages : 3

SARDAR PATEL UNIVERSITY

[86]

BSc Examination [Semester: V]

Subject: Physics Course: US05CPHY22

Mathematical Methods

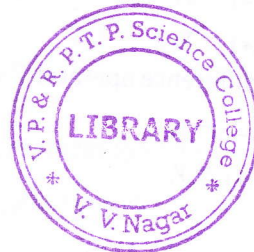
Date: 26-12-2020, Saturday

Time: 02.00 pm to 04.00 pm

Total Marks: 70

## INSTRUCTIONS:

- 1 Attempt all questions.
- 2 The symbols have their usual meaning.
- 3 Figures to the right indicate full marks.



## Q-1 Multiple Choice Questions: [Attempt all]

[10]

(i) The orthogonality condition for curvilinear co-ordinates is \_\_\_\_.

(a)  $\frac{\partial \vec{r}}{\partial u} \cdot \frac{\partial \vec{r}}{\partial v} = 0$

(b)  $\frac{\partial \vec{r}}{\partial v} \cdot \frac{\partial \vec{r}}{\partial v} = 0$

(c)  $\frac{\partial u}{\partial \vec{r}} \cdot \frac{\partial v}{\partial \vec{r}} = 0$

(d)  $\frac{\partial \vec{r}}{\partial u} \cdot \frac{\partial u}{\partial \vec{r}} = 0$

(ii) For curvilinear coordinates  $\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} =$  \_\_\_\_.

(a)  $\frac{h_1 h_3}{h_2} \frac{\partial \vec{r}}{\partial w}$

(b)  $\frac{h_1 h_2}{h_3} \frac{\partial \vec{r}}{\partial w}$

(c)  $\frac{h_1}{h_2 h_3} \frac{\partial \vec{r}}{\partial w}$

(d)  $\frac{h_3 h_2}{h_1} \frac{\partial \vec{r}}{\partial w}$

(iii) For Hermite's function,  $H_0(x) =$  \_\_\_\_.

(a) 0

(b) -1

(c) 1

(d) -4

(iv) For Legendre's equation, \_\_\_\_.

(a)  $k = n$  or  $k = -n - 1$

(b)  $k = n$  or  $k = -n$

(c)  $k = 1$  or  $k = -1$

(d)  $k = n$  or  $k = n - 1$

(v) For Bessel's polynomial, the generating function is given by \_\_\_\_.

(a)  $e^{2tx-t^2}$

(b)  $e^{2x-t^2}$

(c)  $e^{\frac{x}{2}(t^2-1)}$

(d)  $e^{\frac{x}{2}(t-\frac{1}{t})}$

(vi) For a Fourier series of a periodic function  $f(x)$  in  $[-\pi, \pi]$ , the coefficients  $b_n =$  \_\_\_\_.

(a)  $\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$

(b)  $\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$

(c)  $\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$

(d)  $\frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$

(vii) In complex representation of a Fourier series,  $\alpha_n =$  \_\_\_\_.

(a)  $\frac{1}{\tau} \int_0^{\tau} f(t) \cos n\omega t dt$

(b)  $\frac{2}{\tau} \int_0^{\tau} f(t) \cos n\omega t dt$

(c)  $\frac{3}{\tau} \int_0^{\tau} f(t) \cos n\omega t dt$

(d)  $\frac{4}{\tau} \int_0^{\tau} f(t) \cos n\omega t dt$

(viii) The problem of finding an equation of an approximating curve, which passes through as many points as possible is called \_\_\_\_\_.

- (a) Curve fitting (b) Interpolation  
(c) Telegraphy equation (d) Extrapolation

(ix) The forward difference operator  $\Delta$  defined as \_\_\_\_\_.

- (a)  $\Delta y_i = y_{i-1} - y_i$  (b)  $\Delta y_i = y_i - y_{i-1}$   
(c)  $\Delta y_i = y_{i+1} - y_i$  (d)  $\Delta y_i = y_i - y_{i+1}$

(x) The backward difference operator  $\nabla$  defined as \_\_\_\_\_.

- (a)  $\nabla y_i = y_{i-1} - y_i$  (b)  $\nabla y_i = y_i - y_{i-1}$   
(c)  $\nabla y_i = y_{i+1} - y_i$  (d)  $\nabla y_i = y_i - y_{i+1}$

Q-2 State True or False. [Attempt all]

[ 8 ]

- (1) For curvilinear coordinates  $ds^2 = h_1^2 du^2 + h_2^2 dv^2 + h_3^2 dw^2$ .  
(2) For the cylindrical system the unit vectors are  $\hat{e}_r, \hat{e}_\theta$  and  $\hat{e}_\phi$ .  
(3)  $J_n(\mu)$  is the coefficient of  $h^n$  in the expansion of  $(1 - 2\mu h + h^2)^{-1/2}$ .  
(4)  $(1 - x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + n(n+1)y = 0$  is a Bessel's differential equation.  
(5) The phase angle is given by  $\phi_n = \tan^{-1} \left( \frac{\beta_n}{\alpha_n} \right)$ .  
(6) The sine series for  $f(x)$  is given by  $\frac{2}{\pi} \sum_{n=1}^{\infty} \sin nx \int_0^{\pi} f(\vartheta) \sin n\vartheta d\vartheta$  when  $0 \ll x \ll \pi$ .  
(7) For a function  $y = f(x)$ , for a given table of values  $(x_k, y_k), k = 1, 2, \dots, n$ , the process of estimating the value of  $y$ , for any intermediate value of  $x$  is called interpolation.  
(8) The shift operator  $E$  is defined as  $Ef(x) = f(x+h)$ .

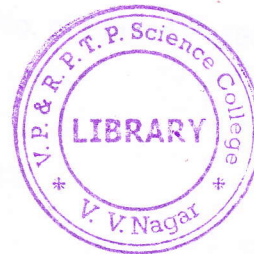


Q-3 Answer the following questions in short. (Attempt any ten)

[ 20 ]

- (1) Define curvilinear coordinates.  
(2) Write down Laplacian in terms of orthogonal curvilinear coordinates.  
(3) If  $u = x + 4, v = y - 2, w = 3z + 1$ , show that  $u, v, w$  are orthogonal.  
(4) Write Hermite's differential equation.  
(5) Using equation:  $H_n(x) = e^{x^2} (-1)^n \frac{d^n e^{-x^2}}{dx^n}$ , find out  $H_1(x)$ .  
(6) Show that  $P_n(-\mu) = (-1)^n P_n(\mu)$ .  
(7) Write cosine series for  $f(x)$  when  $0 \leq x \leq \pi$ . (Note: derivation is not required)

- (8) Write one dimensional wave equation.
- (9) Write telegraphy equation.
- (10) Define interpolation.
- (11) Derive an equivalent equation of a straight line for  $y = ax^b$ .
- (12) For a shift operator  $E$ , show that  $E = \Delta + 1$ .



**Q.4 Long Answer Questions. (Attempt any four)**

[32]

- (1) Derive expression of gradient in terms of orthogonal curvilinear system.
- (2) Derive expression of curl in terms of orthogonal curvilinear system.
- (3) Derive the series solution of Legendre differential equation in the form of descending power of  $x$ .
- (4) Derive the series solution of Bessel's differential equation in the form of ascending power of  $x$ .
- (5) Write the Fourier series for a periodic function  $f(x)$  defined in the interval  $[-\pi, \pi]$ . Derive the coefficients  $a_0, a_n$  and  $b_n$  of the series.
- (6) Derive the Fourier series for a complex periodic function  $f(t)$  defined in  $(-\infty, \infty)$ . Also find the coefficients  $\alpha_n$  and  $\beta_n$ .
- (7) Derive Lagrange's interpolation formula.
- (8) Using the method of least squares, find the straight line  $y = ax + b$  that fits the following data.

$x$	0.5	1.0	1.5	2.0	2.5	3.0
$y$	15	17	19	14	10	7

Use the normal equations of least square fitting that fits a straight line  $y = ax + b$ :

$$a \sum_{i=1}^n x_i^2 + b \sum_{i=1}^n x_i = \sum_{i=1}^n x_i y_i, \quad a \sum_{i=1}^n x_i + bn = \sum_{i=1}^n y_i$$

————— X' —————