

SEAT No. _____

[106]

SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER-V) EXAMINATION-2020
December 28, 2020, Monday
2:00 p.m. to 4:00 p.m.
US05CMTH23 (Group Theory)

Maximum Marks: 70

Q.1 Write the correct option of the following multiple choice questions. [10]

- (1) Multiplicative inverse of $\sqrt{7}$ in \mathbb{R}^* is
(a) $-\sqrt{7}$ (b) $\frac{1}{\sqrt{7}}$ (c) 0 (d) 1
- (2) In Klein 4-group $G = \{e, a, b, c\}$, $abc = \dots\dots\dots$
(a) c (b) e (c) b (d) a
- (3) A nonempty subset H of finite group $(G, +)$ is a subgroup of G iff
(a) $a + b \in H$ (b) $a + b \in G$ (c) $ab^{-1} \in H$ (d) $ab \in G$
- (4) $\phi(11) = \dots\dots\dots$
(a) 10 (b) 11 (c) 1 (d) 0
- (5) If H and K are finite subgroup of group G such that $(o(H), o(K)) = 1$, then $H \cap K = \dots\dots\dots$
(a) $\{e\}$ (b) e (c) ϕ (d) 1
- (6) Let $H = 4Z$ $G = Z$ then $H - 1 = \dots\dots\dots$
(a) $H + 1$ (b) $H + 4$ (c) $H - 3$ (d) $H + 3$
- (7) Homomorphic image of abelian group is
(a) simple (b) cyclic (c) abelian (d) 2
- (8) A homomorphism f is iff $\text{Ker } f = \{e\}$.
(a) one-one (b) onto (c) isomorphism (d) automorphism
- (9) Order of S_5 is
(a) 4 (b) 5 (c) 24 (d) 120
- (10) The group A_n is simple for
(a) $n \geq 4$ (b) $n \geq 3$ (c) $n \geq 5$ (d) $n \geq 1$



Q.2 Do as directed. [08]

- (1) Multiplicative inverse of 2 in Z_7^* is
- (2) True or False?
If $a * e = a = e * a$ for $a, e \in G$ then $(G, *)$ is called semi group.
- (3) In any group G, $o(e) = \dots\dots\dots$
- (4) True or False?
Every group of order 4 is abelian group.
- (5) True or False?
Every subgroup of cyclic group is normal subgroup.
- (6) Define $f : R^* \rightarrow R^*$ by $f(x) = 1/x$ then $\text{Ker } f = \dots\dots\dots$
- (7) True or False?
 S_n/A_n is cyclic group.
- (8) Number of conjugate classes of S_3 is

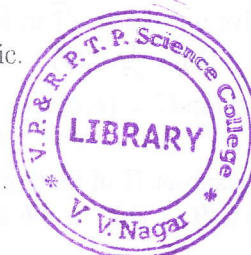
[1]

(P.T.O.)

Q.3 Answer the following in short. (Attempt any 10)

[20]

- (1) Prove that every group has unique identity element.
- (2) Prove that group G , prove that every element of G has unique inverse.
- (3) For group G prove that $(ab)^{-1} = b^{-1}a^{-1}$
- (4) Find all right cosets of $-3\mathbb{Z}$ in \mathbb{Z} .
- (5) Prove that any infinite cyclic group is isomorphic to \mathbb{Z} .
- (6) Let H be any subgroup of group G . Then prove that $aH = H \Leftrightarrow a \in H$.
- (7) Define: Group homomorphism.
- (8) Prove that homeomorphic image of cyclic group is also cyclic.
- (9) Define: Direct sum of subgroups.
- (10) Explain Signature of permutation with example.
- (11) Prove that S_n is a finite non commutative group of order $n!$.
- (12) Find all Sylow 2 - subgroups of S_3 .



Q.4 Answer the following questions. (Attempt any 4)

[32]

- (1) Let H and K be subgroups of group G . Then prove that HK is subgroup of G iff $HK = KH$.
- (2) Let G be a cyclic group and H be a subgroup of G . Then prove that H is cyclic.
- (3) Prove that every infinite cyclic group has only one non-trivial automorphism.
- (4) Let $G = \langle a \rangle$ be a finite cyclic group of order n . Then prove that the mapping $\theta : G \rightarrow G$ defined by $\theta(a) = a^m$ is an automorphism of G iff m is relatively prime to n .
- (5) State and prove First isomorphism theorem.
- (6) Prove that G is direct product of subgroups H and K iff (i) every $x \in G$ can be uniquely expressed as $x = hk$, $h \in H$, $k \in K$ (ii) $hk = kh$, $h \in H$, $k \in K$
- (7) State and prove Cayley's theorem.
- (8) State and prove Sylow Theorem I.

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[2]