No. of Printed Pages : 2

SEAT No.________

[106]

SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER-V) EXAMINATION-2020
December 28, 2020, Monday
2:00 p.m. to 4:00 p.m.
US05CMTH23 (Group Theory)

Maximum Marks: 70

Q.1 Write the correct option of the following multiple choice questions. [10]

(1) Multiplicative inverse of $\sqrt{7}$ in $\mathbb{R}^*$ is ....
(a) $-\sqrt{7}$ (b) $\frac{1}{\sqrt{7}}$ (c) 0 (d) 1

(2) In Klein 4-group $G = \{e, a, b, c\}$, $abc =$ ....
(a) c (b) e (c) b (d) a

(3) A nonempty subset H of finite group $(G, +)$ is a subgroup of G iff ....
(a) $a + b \in H$ (b) $a + b \in G$ (c) $ab^{-1} \in H$ (d) $ab \in G$

(4) $\phi(11) =$ ....
(a) 10 (b) 11 (c) 1 (d) 0

(5) If H and K are finite subgroup of group G such that $((o(H)) , O(K)) = 1$,then $H \cap K =$ ....
(a) $\{e\}$ (b) e (c) ϕ (d) 1

(6) Let $H = 4Z$ $G = Z$ then $H - 1 =$ ....
(a) $H + 1$ (b) $H + 4$ (c) $H - 3$ (d) $H + 3$

(7) Homomorphic image of abelian group is ....
(a) simple (b) cyclic (c) abelian (d) 2

(8) A homomorphism f is .... iff $Kerf = \{e\}$.
(a) one-one (b) onto (c) isomorphism (d) automorphism

(9) Order of S_5 is ....
(a) 4 (b) 5 (c) 24 (d) 120

(10) The group A_n is simple for ....
(a) $n \geq 4$ (b) $n \geq 3$ (c) $n \geq 5$ (d) $n \geq 1$

Q.2 Do as directed. [08]

(1) Multiplicative inverse of 2 in $Z_7{}^*$ is ....

(2) **True or False?**
If $a * e = a = e * a$ for $a, e \in G$ then $(G, *)$ is called semi group.

(3) In any group G, $o(e) =$ ....

(4) **True or False?**
Every group of order 4 is abelian group.

(5) **True or False?**
Every subgroup of cyclic group is normal subgroup.

(6) Define $f : R^* \to R^*$ by $f(x) = 1/x$ then Ker $f =$ ....

(7) **True or False?**
S_n/A_n is cyclic group.

(8) Number of conjugate classes of S_3 is ....

(P.T.O.)

Q.3 Answer the following in short. (Attempt any 10) [20]

(1) Prove that every group has unique identity element.

(2) Prove that group G, prove that every element of G has unique inverse.

(3) For group G prove that $(ab)^{-1} = b^{-1}a^{-1}$

(4) Find all right cosets of $-3\mathbb{Z}$ in $\mathbb{Z}$.

(5) Prove that any infinite cyclic group is isomorphic to $\mathbb{Z}$.

(6) Let H be any subgroup of group G. Then prove that $aH = H \Leftrightarrow a \in H$.

(7) Define: Group homomorphism.

(8) Prove that homeomorphic image of cyclic group is also cyclic.

(9) Define: Direct sum of subgroups.

(10) Explain Signature of permutation with example.

(11) Prove that S_n is a finite non commutative group of order $n!$.

(12) Find all Sylow 2 - subgroups of S_3.

Q.4 Answer the following questions. (Attempt any 4) [32]

(1) Let H and K be subgroups of group G. Then prove that HK is subgroup of G iff $HK = KH$.

(2) Let G be a cyclic group and H be a subgroup of G. Then prove that H is cyclic.

(3) Prove that every infinite cyclic group has only one non-trivial automorphism.

(4) Let $G = (a)$ be a finite cyclic group of order n. Then prove that the mapping $\theta : G \to G$ defined by $\theta(a) = a^m$ is an automorphism of G iff m is relatively prime to n.

(5) State and prove First isomorphism theorem.

(6) Prove that G is direct product of subgroups H and K iff (i) every $x \in G$ can be uniquely expressed as $x = hk$, $h \in H$, $k \in K$ (ii) $hk = kh$, $h \in H$, $k \in K$

(7) State and prove Cayley's theorem.

(8) State and prove Sylow Theorem I.

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