SARDAR PATEL UNIVERSITY

B.Sc. (IV-Semester) EXAMINATION 2022

Monday, 11thApril

03:00pm-05:00pm

US04CMTH22-Mathematics

Partial Differential Equations

No. of pages: 03

Total Marks: 70

Note: Figures to the right indicates full marks of question.

Q: 1 Answer the following by selecting the correct answer from the given options: **[10]**

1. Two linearly independent solutions of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ represents--------

   a. family of surfaces b. family of curves c. a point in space d. none of these

2. The solution of the Pfaffian differential equation $2ydx + 2xdy - dz = 0$ is----------

   a. $x^2 + y^2 - 3z^2 = c$ b. $2xy - z^2 = c$ c. $2xy - z = c$ d. $2x^3 - 3z^2 = c$

3. Which of the following is not a solution of : $\frac{xdx}{y^2z} = \frac{dy}{xz} = \frac{dz}{xy^2}$ ---------

   a. $x^3 - y^3 = c$ b. $2x^3 - 3z^2 = c$ c. $2y^3 - 3z^2 = c$ d. $x - 3y = c$

4. Degree of the P.D.E. $\frac{\partial^2 z}{\partial x^2} + x^2 \left(\frac{\partial^2 z}{\partial y^2}\right)^3 + z^2 \left(\frac{\partial z}{\partial y}\right)^5 = 0$ is------

   a. 5 b. 2 c. 3 d. 4

5. Let $F(u, v) = 0$ where $u \equiv y - x = c_1$ and $u \equiv z - x = c_2$ be general solution of $p+q=1$ then solution passing through curve $x = 0, y^2 = z$ is

   a.$(y - x)^2 = z$ b. $(y - x)^2 = z - x$ c. $(z - x)^2 = y - x$ d. none of these

6. Partial differential equation for $z = f(x - y)$ is-------------

   a.$z = x - y$ b. $p = q$ c. $p^2 - q = 0$ d. $p + q = 0$

7. Which of the following is non-linear partial differential equation?

   a.$p - q = z$ b. $xp - yq = z$ c. $pq = z$ d. $x^2p + yq = z$

8. In Charpit's method equation involving only p and q,then Charpit's equation is of the form

   a.$\frac{dp}{x} = \frac{dq}{y}$ b. $\frac{dp}{0} = \frac{dq}{0}$ c. $\frac{dp}{f_x} = \frac{dq}{f_y}$ d. $\frac{dx}{0} = \frac{dy}{0}$

9. For linear P.D.E. with constant co-efficient $F(D, D') = f(x, y)$ the operator $D' = $-----------

   a.$\frac{\partial}{\partial x}$ b. $\frac{\partial}{\partial p}$ c. $\frac{\partial}{\partial q}$ d. $\frac{\partial}{\partial y}$

10. In second order P.D.E.of type $z = f(u) + g(v) + w$, u,v,w are functions of x&y then $s=$---

    a.$\frac{\partial^2 z}{\partial x \partial y}$ b. $\frac{\partial^2 z}{\partial y^2}$ c. $\frac{\partial y}{\partial x}$ d. $\frac{\partial^2 z}{\partial x^2}$

Q:2 Do as Directed:

1. Solution of $\frac{dx}{a^2} = \frac{dy}{b^2} = \frac{dz}{c^2}$ is given by ----------

2. Two systems of curves are said be orthogonal if each curve of one family cut each curve of the other family at angle ----------

3. True or False: In partial differential equation $\frac{\partial z}{\partial x}$ is denoted by q.

4. True or False: Higher order P.D.E. for a function $z = f(x, y)$ is given with relation $p\frac{\partial(u,v)}{\partial(y,z)} + q\frac{\partial(u,v)}{\partial(z,x)} = \frac{\partial(u,v)}{\partial(x,y)}$ where u, v are function of x, y, z.

5. True or False: A complete integral of the P.D.E. $z = px + qy + p + q$ is . $z = ax + by$.

6. True or False: The system of equations $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ are said to be compatible if $[f, g] = 0$.

7. The general form of second order partial differential equation is ------------

8. The equation $z = px + qy + f(p, q)$ is said to be in ------------ form.

Q:3 Answer in brief of the following questions. *(Any Ten)*

1. Solve: $\frac{dx}{y^2(x-y)} = -\frac{dy}{x^2(x-y)} = \frac{dz}{z(x^2+y^2)}$

2. Determine whether Pfaffian differential equation $a^2y^2z^2dx + b^2x^2z^2dy + c^2x^2y^2dz = 0$ is integrable?

3. Solve: $\frac{dx}{y+z} = \frac{dy}{x+z} = \frac{dz}{x+y}$

4. Eliminate a and b from $ax^2 + by^2 + z^2 = 1$.

5. Obtain partial differential equation for a set of sphere having radius λ and centre on XY-plane.

6. Find integral surface of $x^2 + y = c_1$, $xz + y = c_2$ passes through the line $x = 0, y = 1$.

7. Verify that $z = \sqrt{2x + a} + \sqrt{2y + b}$ is the complete integral of $z = \frac{1}{p} + \frac{1}{q}$

8. Find Complete integral of the equation $p + q = pq$

9. Explain non-linear P.D.E. with suitable example.

10. Solve: $r = a^2 t$

11. Solve: $\left(4D^2 + 12DD' + 9D'^2\right)z = 0$

12. Find P.I. of the equation $(D^2 + D')z = 2y - x^2$.

Q:4 Attempt any Four of the following. [32]

1. If X is a vector such that $X.curlX = 0$ and μ is an arbitrary function of x,y,z then $(\mu X).curl(\mu X) = 0$.

2. Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 tan^2 \propto$ at its intersection with the family of planes parallel to $z = 0$.

3. Find the general solution of the linear P.D.E. $px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$

4. Find the general integral of the linear P.D.E $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ and also find the particular integral which passes through the line $x = 1, y = 0$.

5. Show that the equations $xp - yq = x, x^2p + q = xz$ are compatible and find its solution.

6. Show that surface of the equation $(x - a)^2 + (y - b)^2 + z^2 = 1$ is a complete integral of non-linear partial differential equation $z^2(p^2 + q^2 + 1) = 1$. Determine the general solution by finding envelope of its particular solution.

7. If $u_1, u_2, u_3, - - - - -u_n$ are solution of homogeneous linear partial differential equation $F(D,D')z = 0$ then summation $\sum_{r=1}^{n} c_r u_r$ is also a solution of $(D,D')z = 0$.

8. Find the general solution of $(D^2 - DD')z = cosx.cos2y$

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