SEAT No.________

[130]

No of printed page : 2

SARDAR PATEL UNIVERSITY

B.Sc.(SEMESTER - IV) EXAMINATION - 2022

Saturday, 9^{th} April 2022 MATHEMATICS: US04CMTH21

(Ordinary Differential Equations)

Time : 03:00 p.m. to 05:00 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. [10]

(1) Order and degree of $y - xy_1 = [1 + y_1^2]^{5/2}$ are .... respectively .
(a) 1 and 5 (b) 1 and 2 (c) 1 and 10 (d) 1 and 5/2

(2) The integrating factor of the differential equation $(1+x^2)y_1 + y = \tan^{-1} x$ is ....
(a) $e^{\tan x}$ (b) $e^{\tan^{-1} x}$ (c) $e^{-\tan x}$ (d) $\tan x$

(3) $\frac{1}{D-i}\, x =$ ....
(a) $e^{-x} \int xe^{x}\, dx$ (b) $e^{x} \int xe^{x}\, dx$ (c) $e^{ix} \int xe^{-ix}\, dx$ (d) $e^{-ix} \int xe^{ix}\, dx$

(4) $\frac{1}{f(D)}\, e^{2x} \cos 3x =$ ....
(a) $e^{2x}\frac{1}{f(D+2)} \cos 3x$ (b) $e^{2x}\frac{1}{f(D-2)} \cos 3x$ (c) $e^{2x}\frac{1}{f(D+3)} \cos 3x$ (d) $e^{2x}\frac{1}{f(D-3)} \cos 3x$

(5) $\frac{1}{(D^2+9)} \cos 3x =$ ....
(a) $\frac{x}{6} \sin 3x$ (b) $\frac{-x}{6} \sin 3x$ (c) $\frac{x}{6} \cos 3x$ (d) $\frac{-x}{6} \cos 3x$

(6) If $L^{-1}\{f(s)\} = f(t)$, then $L^{-1}\{\overline{f}(s-a)\}$=......
(a) $e^{at}f'(t)$ (b) $e^{at}f(t)$ (c) $f(t)$ (d) None

(7) Inverse of Laplace transforms of $\frac{e^{-2s}}{s-3}$ is ....
(a) $e^{(t-2)}\, u(t-2)$ (b) $e^{3(t-2)}\, u(t-2)$ (c) $-e^{3(t-2)}\, u(t-2)$ (d) $2e^{3(t-2)}\, u(t-2)$

(8) Differential equation of $ay^2 = x^3$ is ....
(a) $3xy_1 = 2y$ (b) $2yy_1 = 3x$ (c) $2xy_1 = 3y$ (d) $2ayy_1 = 3x^2$

(9) For continuous compounding A = ...
(a) e^{rt} (b) Pe^{rt} (c) $-Pe^{rt}$ (d) Pe^{-rt}

(10) Orthogonal trajectories of $y^2 = 4a(x+a)$ is ....
(a) $4c(x+c)$ (b) $4(x+c)$ (c) $4a(x+c)$ (d) $4c(x+a)$

Que.2 Write TRUE or FALSE. [8]

(1) Solution of $p - y = 0$ is $y = c \log x$

(2) The general solution of the differential equation $y = px + \frac{3}{p}$ is $cy = c^2x + 3$

(3) The particular integral of $(D-m)^r y = e^{mx}$ is $\frac{x^r}{r!}e^{mx}$

(4) The complementary function of $(D^3 - D)y = e^x + e^{-x}$ is $c_1 + c_2e^x + c_3xe^{-x}$

(5) $L^{-1}\left[\frac{1}{s^2-a^2}\right] = \frac{1}{a}\sinh at$

(6) If $f'(t)$ is continuous and L{f(t)}=f(s) then $L\{f'(t)\} = s\overline{f}(s) - f(0)$

(P.T.O.)

(7) Orthogonal trajectories for family of circles having centre at origin are ellipses.

(8) Ohm's law is $E = iR^2$

Que.3 Attempt the following (Any TEN) [20]

(1) Solve $(y\cos x + \sin y + y)dx + (\sin x + x\cos y + x)dy = 0$.

(2) Solve $y^2(y - px) = x^4p^2$.

(3) Solve $(x^2 - 2xy - y^2)dx - (x + y)^2dy = 0$.

(4) Let y_1 and y_2 be two solutions of a linear differential equation $\frac{d^ny}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_ny = 0$ and C_1, C_2 be two arbitrary constants. Then prove that $C_1y_1 + C_2y_2$ is also a solution .

(5) Find P.I.(particular integral) of $(2D^2 - 4D + 5)y = e^{3x}$.

(6) Find C.F.(complementary function) of $(D^3 - 3D^2 + 2D)y = \cos 2x$.

(7) Find Laplace transform of $4t^2 + \sin 3t + e^{2t}$

(8) State Linearity Property and First shifting Property.

(9) Define Unit step function.

(10) State Newton's law of cooling .

(11) If Rs. 10000 is invested at 6 percent per annum, find what amount has accumulated after 6 years if interest is compounded (a) annually (b) quarterly.

(12) State Kirchhoff's second law .

Que.4 Attempt the following (Any FOUR) [32]

(1) Prove that the necessary and sufficient condition for the differential equation $\mathrm{M}dx + \mathrm{N}dy = 0$ to be exact is that $\frac{\partial \mathrm{M}}{\partial y} = \frac{\partial \mathrm{N}}{\partial x}$.

(2) Solve $p^2 - py + x = 0$.

(3) Obtain the particular integral of $f(D)y = \sin mx$, where m is constant.

(4) Solve $(D^2 - 5D + 6)y = 4e^x$ subject to the conditions that $y(0) = y'(0) = 1$. Hence find y(16).

(5) Using Laplace transform , solve the simultaneous equations $\frac{dx}{dt} + y = \sin t$, $\frac{dy}{dt} + x = \cos t$, when $x = 0, y = 2$ for $t = 0$.

(6) Evaluate (i) $L\left\{e^{-4t}\int_0^t t\sin 3t\, dt\right\}$ (ii) $\int_0^\infty \frac{e^{-t}\sin^2 t}{t}\, dt$

(7) Water is heated to the boiling point temperature $100^0\ C$ .It is then removed from the heat and kept in a room which is at a constant temperature of $60^0\ C$ .After 3 minutes , the temperature of the water is $90^0\ C$.(a) Find the temperature of water after 6 minutes .(b) When will the temperature of water be $75^0\ C$ and $61^0\ C$?

(8) Find the family orthogonal to the family $y = ce^{-x}$ of exponential curves.Determine the member of each family passing through (0,4) .

— x —