

SEAT No. _____



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SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - IV) EXAMINATION - 2022
Saturday, 9th April 2022 MATHEMATICS: US04CMTH21
(Ordinary Differential Equations)

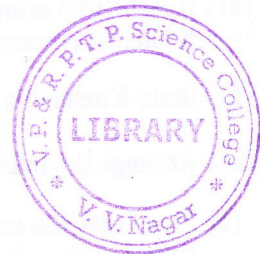
Time : 03:00 p.m. to 05:00 p.m.

Maximum Marks : 70

Que.1 Fill in the blanks.

[10]

- (1) Order and degree of $y - xy_1 = [1 + y_1]^{15/2}$ are respectively .
(a) 1 and 5 (b) 1 and 2 (c) 1 and 10 (d) 1 and 5/2
- (2) The integrating factor of the differential equation $(1+x^2)y_1 + y = \tan^{-1} x$ is
(a) $e^{\tan x}$ (b) $e^{\tan^{-1} x}$ (c) $e^{-\tan x}$ (d) $\tan x$
- (3) $\frac{1}{D-i} x = \dots\dots\dots$
(a) $e^{-x} \int x e^x dx$ (b) $e^x \int x e^x dx$ (c) $e^{ix} \int x e^{-ix} dx$ (d) $e^{-ix} \int x e^{ix} dx$
- (4) $\frac{1}{f(D)} e^{2x} \cos 3x = \dots\dots\dots$
(a) $e^{2x} \frac{1}{f(D+2)} \cos 3x$ (b) $e^{2x} \frac{1}{f(D-2)} \cos 3x$ (c) $e^{2x} \frac{1}{f(D+3)} \cos 3x$ (d) $e^{2x} \frac{1}{f(D-3)} \cos 3x$
- (5) $\frac{1}{(D^2+9)} \cos 3x = \dots\dots\dots$
(a) $\frac{x}{6} \sin 3x$ (b) $\frac{-x}{6} \sin 3x$ (c) $\frac{x}{6} \cos 3x$ (d) $\frac{-x}{6} \cos 3x$
- (6) If $L^{-1}\{f(s)\} = f(t)$, then $L^{-1}\{\bar{f}(s-a)\} = \dots\dots$
(a) $e^{at} f(t)$ (b) $e^{at} f(t)$ (c) $f(t)$ (d) None
- (7) Inverse of Laplace transforms of $\frac{e^{-2s}}{s-3}$ is
(a) $e^{(t-2)} u(t-2)$ (b) $e^{3(t-2)} u(t-2)$ (c) $-e^{3(t-2)} u(t-2)$ (d) $2e^{3(t-2)} u(t-2)$
- (8) Differential equation of $ay^2 = x^3$ is
(a) $3xy_1 = 2y$ (b) $2yy_1 = 3x$ (c) $2xy_1 = 3y$ (d) $2ayy_1 = 3x^2$
- (9) For continuous compounding A =
(a) e^{rt} (b) Pe^{rt} (c) $-Pe^{rt}$ (d) Pe^{-rt}
- (10) Orthogonal trajectories of $y^2 = 4a(x+a)$ is
(a) $4c(x+c)$ (b) $4(x+c)$ (c) $4a(x+c)$ (d) $4c(x+a)$



Que.2 Write TRUE or FALSE.

[8]

- (1) Solution of $p - y = 0$ is $y = c \log x$
- (2) The general solution of the differential equation $y = px + \frac{3}{p}$ is $cy = c^2x + 3$
- (3) The particular integral of $(D - m)^r y = e^{mx}$ is $\frac{x^r}{r!} e^{mx}$
- (4) The complementary function of $(D^3 - D)y = e^x + e^{-x}$ is $c_1 + c_2 e^x + c_3 x e^{-x}$
- (5) $L^{-1} \left[\frac{1}{s^2 - a^2} \right] = \frac{1}{a} \sinh at$
- (6) If $f'(t)$ is continuous and $L\{f(t)\} = f(s)$ then $L\{f'(t)\} = s\bar{f}(s) - f(0)$

(7) Orthogonal trajectories for family of circles having centre at origin are ellipses.

(8) Ohm's law is $E = iR^2$

Que.3 Attempt the following (Any TEN)

(1) Solve $(y \cos x + \sin y + y)dx + (\sin x + x \cos y + x)dy = 0$.

(2) Solve $y^2(y - px) = x^4p^2$.

(3) Solve $(x^2 - 2xy - y^2)dx - (x + y)^2dy = 0$.

(4) Let y_1 and y_2 be two solutions of a linear differential equation $\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + \dots + a_n y = 0$ and C_1, C_2 be two arbitrary constants. Then prove that $C_1 y_1 + C_2 y_2$ is also a solution.

(5) Find P.I.(particular integral) of $(2D^2 - 4D + 5)y = e^{3x}$.

(6) Find C.F.(complementary function) of $(D^3 - 3D^2 + 2D)y = \cos 2x$.

(7) Find Laplace transform of $4t^2 + \sin 3t + e^{2t}$

(8) State Linearity Property and First shifting Property.

(9) Define Unit step function.

(10) State Newton's law of cooling.

(11) If Rs. 10000 is invested at 6 percent per annum, find what amount has accumulated after 6 years if interest is compounded (a) annually (b) quarterly.

(12) State Kirchhoff's second law.

Que.4 Attempt the following (Any FOUR)

(1) Prove that the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact is that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

(2) Solve $p^2 - py + x = 0$.

(3) Obtain the particular integral of $f(D)y = \sin mx$, where m is constant.

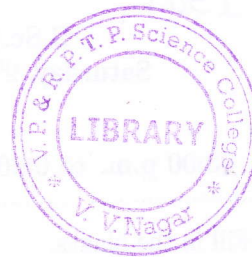
(4) Solve $(D^2 - 5D + 6)y = 4e^x$ subject to the conditions that $y(0) = y'(0) = 1$. Hence find $y(16)$.

(5) Using Laplace transform, solve the simultaneous equations $\frac{dx}{dt} + y = \sin t$, $\frac{dy}{dt} + x = \cos t$, when $x = 0, y = 2$ for $t = 0$.

(6) Evaluate (i) $L \left\{ e^{-4t} \int_0^t t \sin 3t dt \right\}$ (ii) $\int_0^\infty \frac{e^{-t} \sin^2 t}{t} dt$

(7) Water is heated to the boiling point temperature $100^\circ C$. It is then removed from the heat and kept in a room which is at a constant temperature of $60^\circ C$. After 3 minutes, the temperature of the water is $90^\circ C$. (a) Find the temperature of water after 6 minutes. (b) When will the temperature of water be $75^\circ C$ and $61^\circ C$?

(8) Find the family orthogonal to the family $y = ce^{-x}$ of exponential curves. Determine the member of each family passing through $(0,4)$.



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