[84] SEAT No. ________

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Sardar Patel University

B.Sc. Sem:III ,Nomember:2021

Subject : Mathematics **US03CMTH22 [MultivariateCalculus]** **Max.Marks: 70**

Date :30/11/2021 **Time:03.00P.M. TO 05.00 P.M.**

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Q.1 Choose the correct option for each of the following. **[10]**

(1) $\beta(1,1) =$......

(a) 1 (b) 2 (c) 3 (d) None of these

(2) If $\emptyset(x, y, z) = xyz$, the value of $|grad\emptyset|$ at the point (1,2,-1) is

(a) 1 (b) 0 (c) 2 (d) 3

(3) The value of $\int_0^{\pi} \frac{dx}{x^2+4}$ is ....

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

(4) $\int_0^1 \int_0^2 dx\, dy$ = ...

(a) 0 (b) 1 (c) 2 (d) none

(5) $\frac{ds}{dt}$ = ...

(a) $\left|\frac{d\bar{r}}{dt}\right|$ (b) $\frac{d\bar{r}}{dt}$ (c) $\left|\frac{d\bar{r}}{ds}\right|$ (d) $\frac{dr}{dt}$

(6) If $x = r\cos\theta, y = r\sin\theta$ then Jacobian $J =$ ......

(a) 1 (b) r (c) r^2 (d) 2

(7) If $f = -xy^2,\ g = x^2y$ then $\frac{\partial f}{\partial y} - \frac{\partial g}{\partial x}$ = ...

(a) $4xy$ (b) $-4xy$ (c) $2xy$ (d) $-2xy$

(8) $\int_C [f dx + g dy + h dz]$ is independent of path iff fdx+gdy+hdz is ......

(a) 0 (b) not exact (c) 1 (d) exact

(9) $\int_0^1 \int_0^1 \int_0^1 x\,dx\,dy\,dz$ = ...

(a) 1 (b) 0 (c) 2 (d) $\frac{1}{2}$

(10) A function $f(x, y, z)$ is said to be harmonic if $\nabla^2 f =$....

(a) 0 (b) −1 (c) 1 (d) 2

(P.T.O.)

Q.2 Do as Directed: [8]

(1) True or False : $\beta(m,n) = \int_0^{\frac{\pi}{2}} sin^{2m-1}\theta \; cos^{2n-1}\theta \; d\theta.$

(2) True or False : *The vector* $3x^2\bar{\imath} - 4y\bar{\jmath} + z\bar{k}$ is irraotational.

(3) If we change Car tesian variable (x,y) to polar variable (r,θ) then dxdy = ...

(4) True or False : The moment of inertia about origin defined as $I_0 = I_X + I_Y$.

(5) The area of plane region in cartesian form is given by A = ...

(6) If $W = 2x^2 + y^2$ then $\nabla^2 W$ = ...

(7) In Stock's theorem $\iint_S (\bar{\nabla} \times \bar{V}).\bar{n}$ dA = ...

(8) If f is harmonic function then $\iint_S \frac{\partial f}{\partial n}$ dA = ...

Q.3 Attempt any Ten. [20]

(1) Define: A Beta function.

(2) Prove that $\Gamma\frac{1}{2} = \sqrt{\pi}$.

(3) Define: Directional derivatives.

(4) Evaluate the line integral $\int_C (3x^2 + 3y^2)$ ds, where C: over the path y = x from (0,0) to (1,1) (Counterclockwise direction).

(5) Define : Line integral.

(6) Find area of the region bounded by parabola y^2 =4 - x and y^2=4 -4x .

(7) Prove that the form under integral sign in

$$\int_{(1,1,2)}^{(3,-2,-1)} [yzdx + xzdy + xydz]$$ is exact and hence evaluate it.

(8) Find tangent plane to the surface $x^2 + y^2 = z$ at (2,1,5).

(9) Represent the surface $x^2 + y^2 + z^2 = a^2$ in parametric form.

(10) Define: Harmonic function.

(11) Evaluate: $\iint_S [yzdydz + zxdzdx + xydxdy], where\ S: x^2 + y^2 + z^2 = 1.$

(12) If $\bar{V} = \bar{\nabla} f$ then prove that $\int_C \bar{V}_t \, ds = 0$.

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Q.4 Attempt any FOUR. **[32]**

(1) State and Prove relation between Beta and Gamma function.

(2) Let $f(r) = \frac{e^{\lambda r}}{r}$, be a scalar point function then prove that $(\nabla^2 - \lambda^2)f=0$, where $r = \sqrt{x^2 + y^2 + z^2}$.

(3) Evaluate $\iint_R (x^2 + y^2)\, dxdy, where$ R is the parallelogram with vertices R :(0,0),(1,1),(2,0),(1,-1),& $x + y = u, x - y = v$.

(4) Find the volume of the region bounded by the cylinder $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$.

(5) State and Prove Green's theorem for plane.

(6) Verify both vector form (divergence and curl form) of Green's theorem for the given $\bar{V} = 7x\bar{\imath} - 3y\bar{\jmath}$ and C : the circle $x^2 + y^2 = 4$.

(7) State and Prove Divergence theorem of Gauss.

(8) Verify the Stock's theorem for the $\bar{V} = (2x - y)\,\bar{\imath} - y\,z^2\bar{\jmath} - z\,y^2\bar{k}$ and surface S : The upper half surface of the sphere $x^2 + y^2 + z^2 = 1$.

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