

[84]

SEAT No. _____



No. of Printed Pages: 03

Sardar Patel University

B.Sc. Sem:III, Nomenclature:2021

Subject : Mathematics

US03CMTH22 [Multivariate Calculus]

Max.Marks: 70

Date :30/11/2021

Time:03.00P.M. TO 05.00 P.M.

Q.1 Choose the correct option for each of the following.

[10]

(1) $\beta(1,1) = \dots$

- (a) 1 (b) 2 (c) 3 (d) None of these

(2) If $\phi(x, y, z) = xyz$, the value of $|\text{grad}\phi|$ at the point (1,2,-1) is

- (a) 1 (b) 0 (c) 2 (d) 3

(3) The value of $\int_0^\pi \frac{dx}{x^2+4}$ is

- (a) 0 (b) 1 (c)
- $\frac{\pi}{2}$
- (d)
- $\frac{\pi}{4}$

(4) $\int_0^1 \int_0^2 dx dy = \dots$

- (a) 0 (b) 1 (c) 2 (d) none

(5) $\frac{ds}{dt} = \dots$

- (a)
- $\left| \frac{d\vec{r}}{dt} \right|$
- (b)
- $\frac{d\vec{r}}{dt}$
- (c)
- $\left| \frac{d\vec{r}}{ds} \right|$
- (d)
- $\frac{dr}{dt}$

(6) If $x = r\cos\theta$, $y = r\sin\theta$ then $Jacobian J = \dots$

- (a) 1 (b) r (c)
- r^2
- (d) 2

(7) If $f = -xy^2$, $g = x^2y$ then $\frac{\partial f}{\partial y} - \frac{\partial g}{\partial x} = \dots$

- (a)
- $4xy$
- (b)
- $-4xy$
- (c)
- $2xy$
- (d)
- $-2xy$

(8) $\int_C [f dx + g dy + h dz]$ is independent of path iff $fdx + gdy + h dz$ is

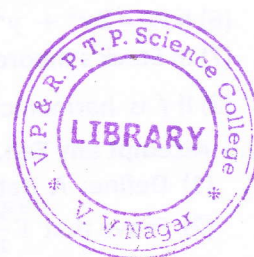
- (a) 0 (b) not exact (c) 1 (d) exact

(9) $\int_0^1 \int_0^1 \int_0^1 x dx dy dz = \dots$

- (a) 1 (b) 0 (c) 2 (d)
- $\frac{1}{2}$

(10) A function $f(x, y, z)$ is said to be harmonic if $\nabla^2 f = \dots$

- (a) 0 (b) -1 (c) 1 (d) 2



(1)

(P.T.O.)

Q.2 Do as Directed:

[8]

- (1) True or False : $\beta(m, n) = \int_0^{\pi} \sin^{2m-1} \theta \cos^{2n-1} \theta \, d\theta$.
- (2) True or False : The vector $3x^2\bar{i} - 4y\bar{j} + z\bar{k}$ is irrotational.
- (3) If we change Cartesian variable (x, y) to polar variable (r, θ) then $dx dy = \dots$
- (4) True or False : The moment of inertia about origin defined as $I_0 = I_x + I_y$.
- (5) The area of plane region in cartesian form is given by $A = \dots$
- (6) If $W = 2x^2 + y^2$ then $\nabla^2 W = \dots$
- (7) In Stock's theorem $\iint_S (\nabla \times \vec{V}) \cdot \vec{n} \, dA = \dots$
- (8) If f is harmonic function then $\iint_S \frac{\partial f}{\partial n} \, dA = \dots$

Q.3 Attempt any Ten.

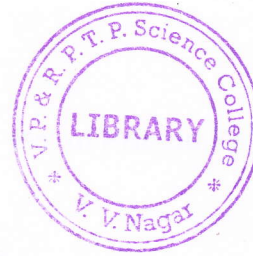
[20]

- (1) Define: A Beta function.
- (2) Prove that $\Gamma \frac{1}{2} = \sqrt{\pi}$.
- (3) Define: Directional derivatives.
- (4) Evaluate the line integral $\int_C (3x^2 + 3y^2) \, ds$, where
C: over the path $y = x$ from $(0,0)$ to $(1,1)$ (Counterclockwise direction).
- (5) Define : Line integral.
- (6) Find area of the region bounded by parabola $y^2 = 4 - x$ and $y^2 = 4 - 4x$.
- (7) Prove that the form under integral sign in

$$\int_{(1,1,2)}^{(3,-2,-1)} [yzdx + xzdy + xydz] \quad \text{is exact and hence evaluate it.}$$

- (8) Find tangent plane to the surface $x^2 + y^2 = z$ at $(2, 1, 5)$.
- (9) Represent the surface $x^2 + y^2 + z^2 = a^2$ in parametric form.
- (10) Define: Harmonic function.
- (11) Evaluate: $\iint_S [yzdydz + xzdzdx + xydx dy]$, where $S: x^2 + y^2 + z^2 = 1$.
- (12) If $\vec{V} = \nabla f$ then prove that $\int_C \vec{V}_t \, ds = 0$.





Q.4 Attempt any FOUR.

[32]

- (1) State and Prove relation between Beta and Gamma function.
- (2) Let $f(r) = \frac{e^{\lambda r}}{r}$, be a scalar point function then prove that $(\nabla^2 - \lambda^2)f=0$, where $r = \sqrt{x^2 + y^2 + z^2}$.
- (3) Evaluate $\iint_R (x^2 + y^2) dx dy$, where R is the parallelogram with vertices $R : (0,0), (1,1), (2,0), (1,-1)$, & $x + y = u, x - y = v$.
- (4) Find the volume of the region bounded by the cylinder $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$.
- (5) State and Prove Green's theorem for plane.
- (6) Verify both vector form (divergence and curl form) of Green's theorem for the given $\vec{V} = 7x\vec{i} - 3y\vec{j}$ and C : the circle $x^2 + y^2 = 4$.
- (7) State and Prove Divergence theorem of Gauss.
- (8) Verify the Stock's theorem for the $\vec{V} = (2x - y)\vec{i} - yz^2\vec{j} - zy^2\vec{k}$ and surface S : The upper half surface of the sphere $x^2 + y^2 + z^2 = 1$.

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