[110]

SEAT No._____

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Sardar Patel University, Vallabh Vidyanagar

B.Sc. - Semester-III : Examinations : 2021-22

Subject : Mathematics | US03CMTH21 | Max. Marks : 70

Numerical Methods

Date: 29/11/2021, Monday | Timing: 03.00 pm - 05.00 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. 10

[1] Aitkin's Δ^2-process is used for finding approximate
- [A] derivative of a function
- [B] integral of a function
- [C] root of an equation
- [D] none of these

[2] If $f(x) = 0$ is expressed as $x = \phi(x)$ then for approximation of root of $f(x) = 0$ using Iteration method, one of the necessary conditions for the convergence of a sequence of approximations is that

[A] $|\phi(x)| > 1$ [B] $|\phi(x)| < 1$ [C] $|\phi'(x)| > 1$ [D] $|\phi'(x)| < 1$

[3] Which of the following intervals contains a root of $x^2 - 3x - 4 = 0$?

[A] $[1, 3]$ [B] $[3, 5]$ [C] $[5, 7]$ [D] $[7, 9]$

[4] If $y_5 = 4$, and $y_{15} = 10$ then $E^5 y_{10} =$

[A] 5 [B] 10 [C] 15 [D] 20

[5] $E^{\frac{1}{2}} y_n - E^{-\frac{1}{2}} y_n =$

[A] Δy_n [B] δy_n [C] ∇y_n [D] μy_n

[6] Which of the following is true?

[A] $\Delta y_5 = \nabla y_4$ [B] $\Delta y_5 = \nabla y_5$ [C] $\Delta y_4 = \nabla y_5$ [D] $\Delta y_6 = \nabla y_5$

[7] For the given data

x	$x_0 = 6$	$x_1 = 8$	$x_2 = 10$	$x_3 = 12$
y	10	14	20	30

$[x_2\ x_3] =$

[A] 5 [B] 10 [C] 20 [D] 40

[8] In usual notations, we always have $[x_0, x_1]$ ---- $[x_i, x_0]$.

[A] < [B] > [C] = [D] none

[9] In usual notations, the formula

$$\int_a^b f(x).dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + ... + y_{n-1}) + y_n]$$

is known as
- [A] Simpson's $\frac{1}{3}$ rule
- [B] Simpson's $\frac{3}{8}$ rule
- [C] Trapezoidal rule
- [D] none

(P.T.O)

[10] For using Simpson's $\frac{3}{8}$ rule it is required that the number of sub-intervals be

[A] even [B] odd [C] a multiple of 3 [D] a multiple of 8

Q: 2. In the following, depending on the type of question either fill in the blank or answer whether a statement is true-false. 08

[1] Equation $x^3 + 11x + 20 = 0$ has no roots in the interval $[1, 2]$. (True or False?)

[2] Equation $x^3 + 5x - 2 = 0$ has a root in the interval $[0, 1]$. (True or False?)

[3] If $E^7 y_2 = 10$ then $E^4 y_5 + 1 =$ ------.

[4] If $y_{10} = 7$ then $E^2 y_8 + E^8 y_2 =$ -----.

[5] For

x	1	4	5
y	8	12	15

the divided difference $[x_1, x_2]$ is ---- .

[6] To use Lagrange's Interpolation formula for a data set $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$it is necessary that $x_0, x_1, \ldots, x_n$ are equally spaced.

[7] For approximating integral using Simpson's rule it is necessary to take even number of subintervals. (True or False?)

[8] To use trapezoidal rule the number of subintervals must be even. (True or False?)

Q: 3. Answer any TEN of the following. 20

[1] Express $\sin x = 5(x + 2)$ in the form of $x = \phi(x)$, so that the necessary condition for applying the Iteration method is satisfied.

[2] Find first approximation of a root of $x^3 + 8x - 7 = 0$ using bisection method.

[3] Find an interval containing an initial approximation of $5 \sin x + 3 = 0$.

[4] If $E^{10} y_1 = 20$ then find $E^5 y_6 + E^6 y_5$.

[5] Prove that $(1 + \Delta)(1 - \nabla) = 1$.

[6] Prove that $\nabla = 1 - E^{-1}$.

[7] If $y_1 = 4$, $y_3 = 12$, $y_4 = 19$ and $y_x = 7$ find x. Write the formula you use and also give it's name.

[8] Using Langrage's interpolation formula, find $y(x)$ for the data

x	4	5	7
y	10	-5	2

[9] For the given data

x	5	7	8
y	2	5	6

find $y(6)$.

[10] Given that $\frac{dy}{dx} = x^3 + y$, $y(0) = 1$, determine $y(0.02)$ using Euler's method, taking $h = 0.01$

[11] Using Simpson's $\frac{1}{3}$ rule find $\int_1^7 x dx$, with subintervals of length 1 unit.

[12] Using Trapezoidal rule find $\int_0^3 \cos x dx$, with 3 subintervals of equal lengths.

Q: 4. Attempt ANY FOUR of the following questions. 32

[1] Using Bisection method find a real root of the equation $x^3 - x - 4 = 0$ correct upto three decimal places.

[2] Find a real root of $x^3 + x^2 + 2x - 1 = 0$ by method of False Position correct upto three decimal places.

[3] Using Gauss's forward interpolation formula find $f(32)$, given that $f(25) = 0.2707$, $f(30) = 0.3027$, $f(35) = 0.3386$, $f(40) = 0.3794$

[4] Derive Gauss's Forward interpolation formula for equally spaced values of argument.

[5] Given the set of tabulated points (x,y) which are $(1, -3)$, $(3, 9)$, $(4, 30)$ and $(6, 132)$ obtain the value of y when $x = 2$ using Newton's divided difference formula.

[6] Obtain 1^{st} and 2^{nd} order numerical differentiation formula from Newton's forward difference formula.

[7] Evaluate $\int_1^3 \frac{1}{x} dx$, by using Simpson's $\frac{1}{3}$ rule with 4 strips.

[8] Use Picard's method to approximate y when $x = 0.25$, given that $y(0) = 0$ and $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ correct upto three decimal places.