

[110]

SEAT No. \_\_\_\_\_



No. of Printed Pages : 03

Sardar Patel University, Vallabh Vidyanagar

B.Sc. - Semester-III : Examinations : 2021-22

Subject : Mathematics

US03CMTH21

Max. Marks : 70

Numerical Methods

Date: 29/11/2021, Monday

Timing: 03.00 pm - 05.00 pm

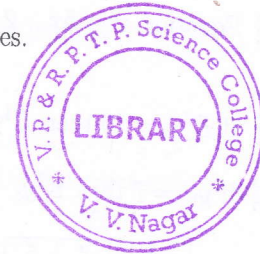
Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices.

10

[1] Aitkin's  $\Delta^2$ -process is used for finding approximate

- [A] derivative of a function  
 [B] integral of a function  
 [C] root of an equation  
 [D] none of these

[2] If  $f(x) = 0$  is expressed as  $x = \phi(x)$  then for approximation of root of  $f(x) = 0$  using Iteration method, one of the necessary conditions for the convergence of a sequence of approximations is that

- [A]  $|\phi(x)| > 1$       [B]  $|\phi(x)| < 1$       [C]  $|\phi'(x)| > 1$       [D]  $|\phi'(x)| < 1$

[3] Which of the following intervals contains a root of  $x^2 - 3x - 4 = 0$ ?

- [A] [1, 3]      [B] [3, 5]      [C] [5, 7]      [D] [7, 9]

[4] If  $y_5 = 4$ , and  $y_{15} = 10$  then  $E^5 y_{10} =$ 

- [A] 5      [B] 10      [C] 15      [D] 20

[5]  $E^{\frac{1}{2}} y_n - E^{-\frac{1}{2}} y_n =$ 

- [A]  $\Delta y_n$       [B]  $\delta y_n$       [C]  $\nabla y_n$       [D]  $\mu y_n$

[6] Which of the following is true?

- [A]  $\Delta y_5 = \nabla y_4$       [B]  $\Delta y_5 = \nabla y_5$       [C]  $\Delta y_4 = \nabla y_5$       [D]  $\Delta y_6 = \nabla y_5$

[7] For the given data

x	$x_0 = 6$	$x_1 = 8$	$x_2 = 10$	$x_3 = 12$
y	10	14	20	30

 $[x_2 \ x_3] =$ 

- [A] 5      [B] 10      [C] 20      [D] 40

[8] In usual notations, we always have  $[x_0, x_1] \dots [x_1, x_0]$ .

- [A] <      [B] >      [C] =      [D] none

[9] In usual notations, the formula

$$\int_a^b f(x).dx = \frac{h}{2} [y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n]$$

is known as

- [A] Simpson's  $\frac{1}{3}$  rule      [B] Simpson's  $\frac{3}{8}$  rule  
 [C] Trapezoidal rule      [D] none

①

(P.T.O)

- [10] For using Simpson's  $\frac{3}{8}$  rule it is required that the number of sub-intervals be  
 [A] even [B] odd [C] a multiple of 3 [D] a multiple of 8

Q: 2. In the following, depending on the type of question either fill in the blank or answer whether a statement is true-false. 08

[1] Equation  $x^3 + 11x + 20 = 0$  has no roots in the interval  $[1, 2]$ . (True or False?)

[2] Equation  $x^3 + 5x - 2 = 0$  has a root in the interval  $[0, 1]$ . (True or False?)

[3] If  $E^7 y_2 = 10$  then  $E^4 y_5 + 1 = \dots$

[4] If  $y_{10} = 7$  then  $E^2 y_8 + E^8 y_2 = \dots$

[5] For 

x	1	4	5
y	8	12	15

 the divided difference  $[x_1, x_2]$  is  $\dots$

[6] To use Lagrange's Interpolation formula for a data set  $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$  it is necessary that  $x_0, x_1, \dots, x_n$  are equally spaced.

[7] For approximating integral using Simpson's rule it is necessary to take even number of subintervals. (True or False?)

[8] To use trapezoidal rule the number of subintervals must be even. (True or False?)

Q: 3. Answer any TEN of the following. 20

[1] Express  $\sin x = 5(x + 2)$  in the form of  $x = \phi(x)$ , so that the necessary condition for applying the Iteration method is satisfied.

[2] Find first approximation of a root of  $x^3 + 8x - 7 = 0$  using bisection method.

[3] Find an interval containing an initial approximation of  $5 \sin x + 3 = 0$ .

[4] If  $E^{10} y_1 = 20$  then find  $E^5 y_6 + E^6 y_5$ .

[5] Prove that  $(1 + \Delta)(1 - \nabla) = 1$ .

[6] Prove that  $\nabla = 1 - E^{-1}$ .

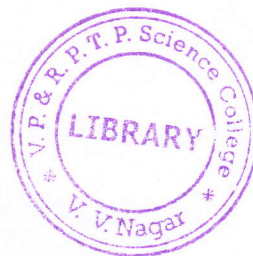
[7] If  $y_1 = 4, y_3 = 12, y_4 = 19$  and  $y_x = 7$  find  $x$ . Write the formula you use and also give its name.

[8] Using Lagrange's interpolation formula, find  $y(x)$  for the data 

x	4	5	7
y	10	-5	2



2



[9] For the given data 

x	5	7	8
y	2	5	6

 find  $y(6)$ .

[10] Given that  $\frac{dy}{dx} = x^3 + y$ ,  $y(0) = 1$ , determine  $y(0.02)$  using Euler's method, taking  $h = 0.01$

[11] Using Simpson's  $\frac{1}{3}$  rule find  $\int_1^7 x dx$ , with subintervals of length 1 unit.

[12] Using Trapezoidal rule find  $\int_0^3 \cos x dx$ , with 3 subintervals of equal lengths.

Q: 4. Attempt ANY FOUR of the following questions.

32

- [1] Using Bisection method find a real root of the equation  $x^3 - x - 4 = 0$  correct upto three decimal places.
- [2] Find a real root of  $x^3 + x^2 + 2x - 1 = 0$  by method of False Position correct upto three decimal places.
- [3] Using Gauss's forward interpolation formula find  $f(32)$ , given that  $f(25) = 0.2707$ ,  $f(30) = 0.3027$ ,  $f(35) = 0.3386$ ,  $f(40) = 0.3794$
- [4] Derive Gauss's Forward interpolation formula for equally spaced values of argument.
- [5] Given the set of tabulated points  $(x,y)$  which are  $(1, -3)$ ,  $(3, 9)$ ,  $(4, 30)$  and  $(6, 132)$  obtain the value of  $y$  when  $x = 2$  using Newton's divided difference formula.
- [6] Obtain 1<sup>st</sup> and 2<sup>nd</sup> order numerical differentiation formula from Newton's forward difference formula.
- [7] Evaluate  $\int_1^3 \frac{1}{x} dx$ , by using Simpson's  $\frac{1}{3}$  rule with 4 strips.
- [8] Use Picard's method to approximate  $y$  when  $x = 0.25$ , given that  $y(0) = 0$  and  $\frac{dy}{dx} = \frac{x^2}{y^2 + 1}$  correct upto three decimal places.

X  
③