

Seat No. : _____

SARDAR PATEL UNIVERSITY

No. of pages: 03

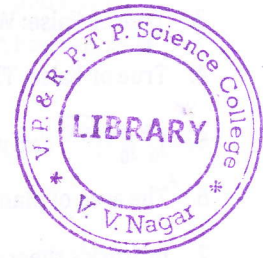
B.Sc. (III-Semester) EXAMINATION 2021

Friday, 1st January, 2021

02:00pm- 04:00pm

US03CMTH 22-Mathematics

MULTIVARIATE CALCULUS



Total Marks: 70

Note: Figures to the right indicates full marks of question.

Q: 1 Answer the following by selecting the correct answer from the given options: [10]

1. If $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ is a constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then $\text{div}(\vec{a} \times \vec{r}) = \dots\dots\dots$
a. 0 b. -1 c. 1 d. a
2. The greatest value of the directional derivative of $f = e^{x+y+z}$ at point (1,0, -1) is -----
a. 1 b. 3 c. 2 d. 4
3. The value of $\int_0^\infty x^2 e^{-x} dx$ is -----
a. 0 b. 2 c. -2 d. ∞
4. $\int_0^3 \int_0^y dx dy = \dots\dots\dots$
a. 3/2 b. -9/2 c. 9/2 d. 3
5. If we change Cartesian variable (x, y) to polar variable (r, θ) in double integration then $dx dy = \dots\dots$
a. $drd\theta$ b. $rdrd\theta$ c. $|J| drd\theta$ d. $r^2 drd\theta$
6. In double integration total mass M of density 1 over region $0 \leq x \leq 2; 0 \leq y \leq 1$ is -----
a. 1 b. 0 c. 4 d. 2
7. The surface $\vec{r} = a\cos v\cos u\vec{i} + a\cos v\sin u\vec{j} + a\sin v\vec{k}$ represents -----
a. sphere b. elliptic paraboloid c. circle d. hyperbolic paraboloid
8. If $yzdx + xzdy + xydz = \dots\dots\dots$
a. $d\left(\frac{x^2+y^2+z^2}{2}\right)$ b. $d(xyz)$ c. $d(x^2 + y^2 + z^2)$ d. $d\left(\frac{xyz}{2}\right)$
9. $\int_0^1 \int_0^1 \int_0^1 xyz dx dy dz = \dots\dots\dots$
a. 8 b. 1/8 c. 1/2 d. 1/4
10. A function $f(x, y, z)$ is said to be harmonic if $\nabla^2 f = \dots\dots$
a. 1 b. 2 c. -1 d. 0

Q: 2 Do as Directed:

[08]

1. True or False: The vector $3x^2\vec{i} - 4y\vec{j} + z\vec{k}$ is irrotational.
2. True or False: If $\phi = xyz$ then value of $|\text{grad}\phi|$ at the point $(1,2,-1)$ is 3.
3. True or False: Work done by force \vec{P} over the curve C is given by $W = \int_C \vec{P}d\vec{r}$
4. True or False: The moment of inertia about origin defined as $I_0 = I_x - I_y$
5. $\int_0^a \int_0^{\sqrt{a^2-x^2}} dx dy = \text{-----}$
6. The area of plane region in polar form is given by $A = \text{-----}$
7. In Stock's theorem $\int \int_S (\vec{v} \times \vec{v}) \cdot \vec{n} dA = \text{-----}$
8. The first form of Green's theorem is $\int \int_R [f\vec{\nabla}^2 g + \vec{\nabla}f \cdot \vec{\nabla}g] dV = \text{-----}$



Q: 3 Answer in brief of the following questions. (Any Ten)

[20]

1. In usual notation prove that $\beta(m, n) = \beta(n, m)$
2. For which value of a the surface $ax^2 - yz = (a+2)x$ and $4x^2y + z^3 - 4 = 0$ are perpendicular to each other at the point $(1, -1, 2)$?
3. Find the equation of tangent plane and normal line to the surface $x^3y^2 - 3x^2z^3 = -zy + 2$ at the point $(0, 2, 1)$.
4. Evaluate $\int_C (y^2 dx - x^2 dy)$ where C: along the straight line from $(0, 1)$ to $(1, 0)$.
5. Evaluate: $\int_0^2 \int_0^y e^{x+y} dx dy$
6. Change the order of integration $\int_0^c \int_0^y f(x, y) dx dy$
7. Verify whether the line integral $\int_{(1,1,2)}^{(3,-2,-1)} (yz dx + xz dy + xy dz)$ is independent of path or not. Also calculate.
8. By using line integral find the area of region $r = a(1 + \cos\theta)$.
9. For which value of u $\vec{r}_u \times \vec{r}_v = 0$? where $\vec{r} = u \cos v \vec{i} + u \sin v \vec{j} + u \vec{k}$.
10. Evaluate $\int \int_S (\sin x dy dz + y(2 - \cos x) dz dx)$ where $S: 0 \leq x \leq 3, 0 \leq y \leq 2, 0 \leq z \leq 1$ by applying divergence theorem.
11. Let R be a closed region in space and S be its boundary. Let g be harmonic function in R, then prove that $\int \int_S \frac{\partial g}{\partial n} dA = 0$
12. If $\vec{v} = \vec{\nabla}f$ then prove that $\int_C \vec{v}_t ds = 0$.

Q: 4 Attempt any Four of the following.

[32]

- (1) State and prove Duplication formula.
- (2) In usual notation prove that: (i) $\text{curl}(r^n \vec{r}) = \vec{0}$
(ii) $\nabla^2(r^n \vec{r}) = n(n+3)r^{n-2}\vec{r}$ Where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$
- (3) Let $f(x, y) = 1$ be the density of mass M in region $R: 0 \leq y \leq \sqrt{1-x^2}, 0 \leq x \leq 1$ then find centre of gravity and moment of inertia I_x, I_y & I_o .
- (4) Transform $\int \int_R (x-y)^2 \sin^2(x+y) dx dy$ in uv -plane by taking $u = x-y, v = x+y$ where R is the parallelogram with vertices $(\pi, 0), (2\pi, \pi), (\pi, 2\pi)$ & $(0, \pi)$. Hence evaluate it.
- (5) State and prove Green's theorem for plane.
- (6) Verify both vector form (divergence and curl form) of Green's theorem for the given \vec{V} and C ,
 $\vec{V} = y\vec{i} + 4x\vec{j}$, C : the boundary of triangle with vertices $(0,0), (2,0)$ & $(2,1)$.
- (7) State and prove Divergence theorem of Gauss.
- (8) Verify the Stock's theorem for $\vec{V} = y^3\vec{i} - x^3\vec{j}$; and surface S : the circular disk $x^2 + y^2 \leq 1, z = 0$

————— X —————

