SEAT No.__________

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SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - II) EXAMINATION - 2022
Wednesday, 27^{th} April 2022 MATHEMATICS: US02CMTH51
(Elementary Algebra)

Time : 12:00 Noon to 02:00 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. (10)

(1) $(cis\theta)^{\frac{21}{9}}$ has only .... distinct value.
(a) 9 (b) 21 (c) 7 (d) 3

(2) $i\tan ix =$................
(a) $\tan x$ (b) $i\tanh x$ (c) $-\tanh x$ (d) $-\tan ix$

(3) If $z = cis\theta$ and $\frac{1}{z} = cis(-\theta)$ then $z^p - \frac{1}{z^p} =$ ....
(a) $2i\sin p\theta$ (b) $-2i\sin p\theta$ (c) $2i\cos p\theta$ (d) 0

(4) Function $f : Z \to Z$ defined by $f(x) = x + 5$ is ....
(a) not oneone (b) not onto (c) bijection (d) None of these

(5) Function $f : A \to A$ is called ....on A .
(a) linear (b) operator (c) matrix (d) None of these

(6) If $(a, m) = d$ then $ax \equiv b(mod\ m)$ has solution iff ....
(a) d/b (b) b/d (c) $d = b$ (d) none

(7) For the system $AX = B$, if the $rank(A) \neq rank(A|B)$ then the system is ....
(a) consistent (b) inconsistent (c) may be both (d) none

(8) If A is a non singular matrix of order n then rank of A is ....
(a) $n-1$ (b) n (c) 1 (d) 0

(9) The matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is ....
(a) Scalar matrix (b) Identity matrix (c) unit matrix (d) None

(10) Characteristic roots of the Identity matrix I of order 2 are ....
(a) $1, -1$ (b) $1, 1$ (c) $0, 1$ (d) $-1, -1$

Que.2 Write TRUE or FALSE. (8)

(1) Amplitude of $-\sqrt{3} + i$ is 150^o .

(2) $\sinh x = -i \sin x$.

(3) Function $f : N \to N$ defined by $f(x) = 2x$ is not onto.

(4) $A \times B$ is trivial relation from A to B.

(5) If A is an orthogonal matrix then A^{-1} is equal to A

(6) If A is a square matrix then $A + A'$ is Symmetric.

(7) The constant term of the characteristics polynomial $|A - xI|$ of A is Adj. A.

(8) The characteristic root of a real Skew- symmetric matrix is either zero or pure imaginary number.

(P. T. O.)

Que.3 Attempt the following (Any TEN) (20)

(1) Find the 7^{th} roots of unity .

(2) Prove that $\sinh^{-1} z = log[z + \sqrt{z^2+1}]$

(3) Separate the real and imaginary parts of $\sin(x+iy)$

(4) Show that the congruence $x + 50 \equiv 39(mod\ 7)$ possesses a solution.

(5) Let $A = \{-2,-1,0,1,2\}$. Let the function $f : A \to R$ is defined by $f(x) = x^2 + 1$. Find the range of f .

(6) Define one one and onto functions.

(7) If A is Hermitian,then prove that $B^{\Theta}AB$ is Hermitian.

(8) If $A = \begin{bmatrix} 1 & 2 \\ -2 & 4 \end{bmatrix}$,$B = \begin{bmatrix} 1 & -3 \\ 2 & 0 \end{bmatrix}$ then check whether $(AB)^T = B^T A^T$.

(9) If A and B are two orthogonal matrices then prove that AB and BA are also orthogonal .

(10) Find Characteristic polynomial of a matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$

(11) Solve the system of equations $x + 3y - 2z = 0;\ \ 2x - y + 4z = 0;\ x - 11y + 14z = 0$

(12) Prove that the characteristic roots of a Hermitian matrix are all real .

Que.4 Attempt the following (Any FOUR) (32)

(1) State and prove De-Moivres theorem .

(2) If $\tan(\theta + i\phi) = e^{i\alpha}$ then prove that $\theta = \left(n + \frac{1}{2}\right)\frac{\pi}{2}$ and $\phi = \frac{1}{2}\log\ \tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$.

(3) Let f be a function defined from the set X to the set Y and let A, B be the subsets of Y, then prove that
(i) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ (ii) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.

(4) If R and S are two equivalence relations on a set A then prove that $R \cap S$ is also an equivalence relation on A .

(5) Using Gauss-Jordan Method find the inverse of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

(6) Convert $A = \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{bmatrix}$ into its equivalent reduced row echelon form and hence find the rank of the matrix A.

(7) State and prove Cayley-Hamilton theorem. Also using it find inverse of non singular matrix.

(8) Find the characteristic roots and corresponding characteristic vectors of $\begin{bmatrix} -2 & -8 & -12 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix}$