

SEAT No. _____



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[68]
E+G

SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - II) EXAMINATION - 2022
Wednesday, 27th April 2022 MATHEMATICS: US02CMTH51
(Elementary Algebra)

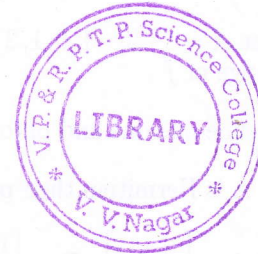
Time : 12:00 Noon to 02:00 p.m.

Maximum Marks : 70

Que.1 Fill in the blanks.

(10)

- (1) $(\cos\theta)^{\frac{21}{9}}$ has only distinct value.
(a) 9 (b) 21 (c) 7 (d) 3
- (2) $i \tan ix = \dots\dots\dots$
(a) $\tan x$ (b) $i \tanh x$ (c) $-\tanh x$ (d) $-\tan ix$
- (3) If $z = \cos\theta$ and $\frac{1}{z} = \cos(-\theta)$ then $z^p - \frac{1}{z^p} = \dots\dots\dots$
(a) $2i \sin p\theta$ (b) $-2i \sin p\theta$ (c) $2i \cos p\theta$ (d) 0
- (4) Function $f : Z \rightarrow Z$ defined by $f(x) = x + 5$ is
(a) not oneone (b) not onto (c) bijection (d) None of these
- (5) Function $f : A \rightarrow A$ is called on A .
(a) linear (b) operator (c) matrix (d) None of these
- (6) If $(a, m) = d$ then $ax \equiv b \pmod{m}$ has solution iff
(a) d/b (b) b/d (c) $d=b$ (d) none
- (7) For the system $AX = B$, if the $\text{rank}(A) \neq \text{rank}(A|B)$ then the system is
(a) consistent (b) inconsistent (c) may be both (d) none
- (8) If A is a non singular matrix of order n then rank of A is
(a) $n-1$ (b) n (c) 1 (d) 0
- (9) The matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is
(a) Scalar matrix (b) Identity matrix (c) unit matrix (d) None
- (10) Characteristic roots of the Identity matrix I of order 2 are
(a) 1, -1 (b) 1, 1 (c) 0, 1 (d) -1, -1



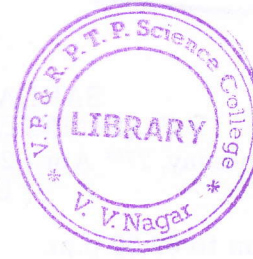
Que.2 Write TRUE or FALSE.

(8)

- (1) Amplitude of $-\sqrt{3} + i$ is 150° .
- (2) $\sinh x = -i \sin x$.
- (3) Function $f : N \rightarrow N$ defined by $f(x) = 2x$ is not onto.
- (4) $A \times B$ is trivial relation from A to B .
- (5) If A is an orthogonal matrix then A^{-1} is equal to A
- (6) If A is a square matrix then $A + A'$ is Symmetric.
- (7) The constant term of the characteristics polynomial $|A - xI|$ of A is $\text{Adj. } A$.
- (8) The characteristic root of a real Skew- symmetric matrix is either zero or pure imaginary number.

(P.T.O.)

Que.3 Attempt the following (Any TEN)



(20)

- (1) Find the 7th roots of unity .
- (2) Prove that $\sinh^{-1} z = \log[z + \sqrt{z^2 + 1}]$
- (3) Separate the real and imaginary parts of $\sin(x + iy)$
- (4) Show that the congruence $x + 50 \equiv 39 \pmod{7}$ possesses a solution.
- (5) Let $A = \{-2, -1, 0, 1, 2\}$. Let the function $f : A \rightarrow R$ is defined by $f(x) = x^2 + 1$. Find the range of f .
- (6) Define one one and onto functions.
- (7) If A is Hermitian, then prove that $B^{\ominus}AB$ is Hermitian.
- (8) If $A = \begin{bmatrix} 1 & 2 \\ -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -3 \\ 2 & 0 \end{bmatrix}$ then check whether $(AB)^T = B^T A^T$.
- (9) If A and B are two orthogonal matrices then prove that AB and BA are also orthogonal .
- (10) Find Characteristic polynomial of a matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$
- (11) Solve the system of equations $x + 3y - 2z = 0$; $2x - y + 4z = 0$; $x - 11y + 14z = 0$
- (12) Prove that the characteristic roots of a Hermitian matrix are all real .

Que.4 Attempt the following (Any FOUR)

(32)

- (1) State and prove De-Moivres theorem .
- (2) If $\tan(\theta + i\phi) = e^{i\alpha}$ then prove that $\theta = \left(n + \frac{1}{2}\right) \frac{\pi}{2}$ and $\phi = \frac{1}{2} \log \tan \left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$.
- (3) Let f be a function defined from the set X to the set Y and let A, B be the subsets of Y , then prove that
(i) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ (ii) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.
- (4) If R and S are two equivalence relations on a set A then prove that $R \cap S$ is also an equivalence relation on A .
- (5) Using Gauss-Jordan Method find the inverse of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$
- (6) Convert $A = \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{bmatrix}$ into its equivalent reduced row echelon form and hence find the rank of the matrix A.
- (7) State and prove Cayley-Hamilton theorem. Also using it find inverse of non singular matrix.
- (8) Find the characteristic roots and corresponding characteristic vectors of $\begin{bmatrix} -2 & -8 & -12 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix}$