SEAT No.________

No of printed page : 2

[116/A-20] Eng

SARDAR PATEL UNIVERSITY

B.Sc.(SEMESTER - I) EXAMINATION - 2022

Monday , 14^{th} February, 2022

MATHEMATICS : US01CMTH51

(Calculus)

Time : 03:00 p.m. to 05:00 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. [10]

(1) $\cosh x + \sinh x =$ ....

(a) 1 (b) e^{x} (c) e^{-x} (d) -1

(2) The n^{th} derivative of $\cos(2x+3)$ is ....

(a) $\sin(2x+3+\frac{\pi}{2})$ (b) $\cos(2x+3+n\frac{\pi}{2})$ (c) $\sin(2x+3+n\frac{\pi}{2})$ (d) None

(3) For the functions u, v, Leibniz's rule give n^{th} derivative of ....

(a) $\dfrac{u}{v}$ (b) uv (c) $\sqrt{uv}$ (d) $u+v$

(4) Asymptotes of $y = x^3 - 3x^2 + 2x$ are ....

(a) x = 0 , 1 , 2 ; y = 1 (b) x = 0 , - 1 , 2 ; y = 0 (c) x = 0 , 1 , - 2 (d) not possible

(5) The curve of $r = 2\theta$ is symmetric about ....

(a) polar axis (b) normal axis (c) pole (d) polar axis , normal axis and pole

(6) $r = \tan\theta \sec\theta$ represent a ....

(a) line (b) parabola (c) ellipse (d) circle

(7) Rectification is a process of ....

(a)Measuring the length of arc on a curve (b)Finding the curvature at a point on the curve

(c)Finding the radius of curvature (d)None of these

(8) Curvature of the line $2x + 3y = 1$ is ....

(a) 0 (b) 2 (c) 1 (d) None

(9) The Euler's theorem is defined for the functions which are.....................

(a) Continuous (b) Differentiable (c) Homogeneous (d) None

(10) If $f(x,y) = \frac{x-y}{x+y}$ then $f_y =$ ....

(a) $\dfrac{2y}{(x+y)^2}$ (b) $\dfrac{-x}{(x+y)^2}$ (c) $\dfrac{-2x}{(x+y)^2}$ (d) $\dfrac{-1}{(x+y)^2}$

Que.2 Write TRUE or FALSE. [8]

(1) The 10^{th} derivative of a^{10x} is $10^{10}(\log 10)\ a^{10x}$.

(2) $\lim\limits_{x\to 0} (\cos x)^{\cot 2x}$ is 0^{∞} form.

(3) The curve $r^2 = 9sin2\theta$ is symmetric about pole only.

(4) The curve of $y = \dfrac{(x-1)(x+2)}{x(x-4)}$ has 3 branches.

P.T.O.

(5) The intrinsic equation is a function of arc length.

(6) For a curve $y = f(x)$ the radius of curvature at a point (x, y) is given by $\frac{(1+{y_1}^2)^{3/2}}{y_2}$.

(7) If $u = f(x-y, y-z, z-x)$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

(8) For $z = \sin^{-1}(\frac{x}{y}) + \tan^{-1}(\frac{y}{x})$. then $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 1$.

Que.3 Attempt the following (**Any TEN**) [20]

(1) Find $\frac{dy}{dx}$ for $y = tan^{-1}(\sinh x)$.

(2) For $y = e^{4x} - log(5x-3)$ find y_2 .

(3) If $y = (ax+b)^{-1}$, then prove that $y_n = \frac{(-1)^n n!\, a^n}{(ax+b)^{n+1}}$.

(4) Find the parametric equation for $x^{2/3} + y^{2/3} = a^{2/3}$.

(5) Express the the point $(3, 40^o)$ in three other ways such that $-2\pi \leqslant \theta \leqslant 2\pi$.

(6) Find tangent parallel to axes for $x = \cos^2\theta$; $y = 2\sin\theta$.

(7) Evaluate $\int_0^{\pi/2} \sin^{10} x\, dx$.

(8) For the curve $y = a\sin 2x$, find $\frac{ds}{dx}$.

(9) Evaluate $\int \sin^4 x \cos^3 x\, dx$.

(10) Is $f(x,y) = \frac{\sqrt[4]{y} + \sqrt[4]{x}}{x-y}$ a homogeneous function ? If so find its degree .

(11) Find $\frac{dy}{dx}$ when $x\sin(x-y) - (x+y) = 0$.

(12) Find $\frac{dz}{dt}$ when $z = sin^{-1}(x-y)$, $x = 3t$ & $y = 4t^3$

Que.4 Attempt the following (**Any FOUR**) [32]

(1) If $x = \cos\left(\frac{1}{m}\log y\right)$, then find $y_n(0)$.

(2) Evaluate $\lim_{x\to 0} \frac{e^x + \log(1-x) - 1}{\tan x - x}$.

(3) Sketch the curve given by $y = \frac{2}{(x+1)(x-2)}$.

(4) Obtain equation of conic, where the directrix is perpendicular as well as parallel to the polar axis and whose one focus is at pole.

(5) Evaluate $\int \cos^7 x\, dx$

(6) Let $r = f(\theta)$ be a polar form of a curve with a point P on it. Then the radius of curvature at P is given by $\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$, where $r_1 = f'(\theta)$ and $r_2 = f''(\theta)$.

(7) State and prove Euler's theorem for $z = f(x, y)$, Also using it prove that $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$.

(8) Let a function y of x be implicitly described by $f(x, y) = c$. Then prove that

(1) $\frac{dy}{dx} = -\frac{f_x}{f_y}$ (2) $\frac{d^2y}{dx^2} = -\frac{f_{xx}(f_y)^2 - 2f_{xy}f_xf_y + f_{yy}(f_x)^2}{(f_y)^3}$.

—X—