Seat No.: [35] Eng.

SARDAR PATEL UNIVERSITY

No. of pages: 2

B.Sc. (I-Semester) EXAMINATION 2021

Saturday, 30th January

02:00pm-04:00pm

US01CMTH 21-Mathematics

CALCULUS

Total Marks: 70

Note: Figures to the right indicates full marks of question.

Q: 1 Answer the following by selecting the correct answer from the given options: **[10]**

1. If $x = cos\left(\frac{1}{m}logy\right)$ then $y(0) =$ -----------

   a. $e^{\frac{m\pi}{2}}$ b. $me^{\frac{m\pi}{2}}$ c. $-me^{\frac{m\pi}{2}}$ d. $e^{m\pi}$

2. The nth derivative of e^{mx} is ------------

   a. e^{mx} b. $m^n e^{mx}$ c. $m^m e^{mx}$ d. $n^m e^{mx}$

3. Vertices of $\frac{x^2}{4} + \frac{y^2}{9} = 1$ are ---------

   a. $(\pm 3, 0)$ b. $(\pm 2, 0)$ c. $(0, \pm 3)$ d. $(0, \pm 2)$

4. $r = cos2\theta$ has --------- loops.

   a. 1 b .5 c.4 d.2

5. Reciprocal equation of $r = \frac{1}{1+cos\theta}$ is

   a. Circle b. Rose Curve c. Cardioid d. Lemniscate

6. $y = x^3 - 3x^2 + 2x = 0$ is symmetry about--------

   a. X-axis b. Y-axis c. Origine d. None of these

7. If $J_n = \int_0^{\frac{\pi}{2}} sin^n x dx$ then J_n =-----

   a. $\frac{n+1}{n} J_{n-2}$ b. $\frac{n-1}{n} J_{n-2}$ c. $\frac{n}{n-1} J_{n-2}$ d. none

8. Volume by cylindrical shell method is V=-----

   a. $2\pi \int_a^b xydx$ b. $\pi \int_a^b xydx$ c. $\pi \int_a^b x^2 dx$ d. None

9. For the curve y=f(x) the radius of curvature at a given point is given by--------

   a. $\frac{(1+{y_1}^2)^{\frac{3}{2}}}{y_2}$ b. $\frac{(r^2+{r_1}^2)^{\frac{3}{2}}}{r^2+2{r_1}^2-rr_2}$ c. $\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ d. none

10. Degree of homogeneity of $u = 3x^2yz + 5xy^2z + 4z^4$ is --------

    a. 2 b. 3 c.4 d.0

(1)

Q: 2 Do as directed: **[08]**

(1) True or False: $\lim_{x\to\frac{\pi}{2}}\left(x-\frac{\pi}{2}\right)tanx$ is $\frac{0}{0}$ form.

(2) True or False: $coshx + sinhx = e^x$

(3) The Cartesian form of a polar equation $r = sec\theta tan\theta$ is ---------

(4) The polar equation of conic, if directrix pass thro' the point $(5, 0^0)$ and e=1 is ---------

(5) Surface area for revolution about X-axis is S=------------

(6) The value of $\int_0^{\frac{\pi}{2}} sin^{10}xdx$ =-----------

(7) True or False: If $u = x^3 - 3xy^2$ then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(8) True or False: The curvature of a straight line is zero.

Q: 3 Answer in brief of the following questions. ***(Any Ten)*** **[20]**

1. Evaluate: $\lim_{x\to 2}\frac{sin(x^2-4)}{x-2}$
2. If $y = e^{3x} - log(7x-5)$ the find y_3.
3. Find centre-to focus distance, foci and center of ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.
4. Express the point $(3, 40^0)$ in three other ways such that $-2\pi \le \theta \le 2\pi$.
5. Discuss asymptotes of Cartesian curve.
6. Find the parametric equation of $\sqrt{x} + \sqrt{y} = \sqrt{a}$
7. Evaluate: $\int sin^4xcos^3xdx$
8. Find the value of θat the point of intersection of $r = a(1 - cos\theta)$ and $r = acos\theta$.
9. Find the area of the surface by revolving the circle $x^2 + y^2 = 1, y > 0$ about X-axis.
10. Find radius of curvature at any point on the curve $s = 8asin^2\left(\frac{\psi}{6}\right)$
11. Find $\frac{dy}{dx}$ when $x^y = y^x$.
12. Verify Euler's theorem for the function $u = \frac{xy}{x+y}$

Q: 4 Attempt any Four of the following: **[32]**

(1) State and Prove Leibniz's theorem.

(2) Evaluate: $\lim_{x\to 0}\frac{e^x+\log(1-x)-1}{tanx-x}$

(3) Sketch: $y = \frac{x(x-4)}{(x-1)(x+2)}$

(4) In usual notation prove that polar equation of a conic is $r = \frac{pe}{1\pm ecos\theta}$

(5) Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} sin^nxdx$, $n \in N$

(6) Find the entire length of asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. Moreover, prove that the length of asteroid measure from $(0, a)$ to the point (x, y) is given by $\frac{3}{2}(ax^2)^{\frac{1}{3}}$.

(7) State and Prove Euler's theorem for z=f(x,y).

(8) Show that the radius of curvature at any point of the curve $x = ae^\theta(cos\theta - sin\theta), y = ae^\theta(cos\theta + sin\theta)$ is twice the perpendicular distance of the tangent at the point from the origin.

—x—