FLOW OF FLUIDS

Chemical engineers are concerned with transportation of fluids, both liquids and gases, from one location to another through pipes or ducts. This activity requires determination of the pressure drop through the system and consequently of the power required for pumping, selection of a suitable type of pumping device and measurement of the flow rates. In this chapter, we will deal with the types of flow patterns, determination of the pressure drop during fluid flow, methods of flow measurements, etc.

7.1 FLUID

7.1.1 Definitions

- A fluid is a substance which is capable of flowing if allowed to do so.
- A fluid is a substance that has no definite shape of its own, but conforms to the shape of the containing vessel.
- A fluid is a substance which when subjected to a shearing force/shear force, however small, undergoes deformation continuously as long as the force is applied.

It can be verified that, liquids and gases/vapours possess the above cited characteristics and hence, are referred to as *fluids*.

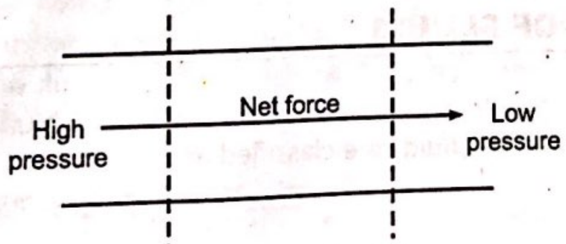
For example, water, ethanol, natural gas, methanol vapours and so on.

7.1.2 Pressure Gradient

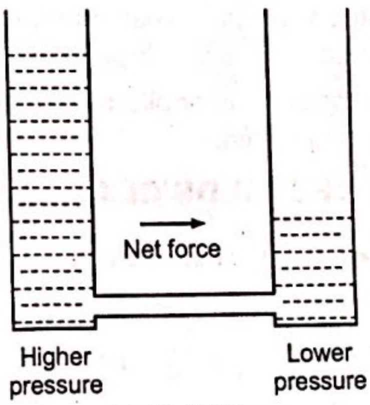
- The change in pressure measured across a given distance is called a pressure gradient.
- The pressure gradient is a dimensional quantity expressed in the units of pressure per unit length. The SI unit of pressure gradient is Pa/m.
- It is the pressure difference in pressure between two locations within a given fluid.

$$\text{Pressure gradient} = \frac{\text{Difference in pressure between two locations}}{\text{Distance between two locations}} = \frac{\Delta P}{\Delta L} \quad \text{or} \quad \frac{\Delta P}{L} \qquad \ldots (7.1)$$

- The pressure gradient results in a net force called the pressure gradient force. This force is directed from high pressure to low pressure within the fluid. The tendency of fluid to flow is from higher pressure to lower pressure. Refer Fig. 7.1 (B).



(A) Pressure gradient force



(B) Fluid flow

Fig. 7.1

7.1.3 Velocity Gradient

The velocity gradient gives an idea of how the velocity of a fluid changes between different points within the fluid.

Velocity gradient is defined *as the difference in velocity between adjacent layers of the fluid.*

The formula for velocity gradient is given as

$$\text{Velocity gradient} = \frac{\Delta u}{\Delta y} \text{ or } \frac{du}{dy} \quad \dots (7.2)$$

where Δu is the difference in flow velocity between the adjacent layers and Δy is the distance between the adjacent layers.

Therefore, the dimensional formula of velocity gradient is $M^0L^0T^{-1}$ and hence its SI unit is 1/s or s^{-1}.

The velocity gradient is also called as rate of shear or rate of shear deformation.

The study of velocity gradient is useful in analysing path dependent materials and further study of stresses and strains. For example, plastic deformation of metals.

7.2 FLUID STATICS AND DYNAMICS : CONCEPT AND APPLICATIONS

- The branch of engineering science which deals with *the behaviour of fluids at rest or in motion* is called as FLUID MECHANICS. The study of water is referred to as Hydraulics.
- Fluid mechanics is classified as : Fluid Statics and Fluid Dynamics.
- **Fluid statics** (hydrostatics) deals with the study of fluids at rest which involves the study of pressure exerted by a fluid at rest and the variation of fluid pressure throughout the fluid.
- **Fluid dynamics** deals with the study of fluids in motion relative to stationary solid walls or boundaries.
- Fluid statics deals with the behaviour of fluids when they are at rest. In fluid statics, there is no relative motion between adjacent fluid layers and hence there is no shear stress in the fluid that deforms it. The only fluid stress is the normal stress due to pressure, so any force developed is due to normal stress, i.e., pressure. Pressure at any point in a fluid is the same in all directions. Pressure varies within a fluid due to the weight of the fluid, i.e., the pressure varies within the fluid along the depth and is equal to ρgh and is the same at any horizontal level in the fluid.
- **Applications of fluid statics :** Submerged bodies such as ships, parts of ships, submarines, dams and gates and liquid storage tanks (to calculate force). It is also used to determine pressures at different levels of atmosphere and in many pressure measuring devices.
- Fluid dynamics is the study of the flow of fluids (liquids and gases) in and around solid surfaces. The fundamental propositions of fluid dynamics are the law of conservation of mass, law of conservation of linear momentum (known as Newton's law of motion) and the law of conservation of energy.
- Applications of fluid dynamics include calculating forces and moments on aircraft, determining the mass flow rates of petroleum through pipelines, predicting weather patterns.
- Some technological applications of fluid dynamics are rocket engines, wind turbines, oil pipelines and air conditioning systems.

7.3 TYPES OF FLUIDS/CLASSIFICATION OF FLUIDS

7.3.1 Ideal and Actual Fluids

Based upon the resistance offered by fluids for their flow, the fluids are classified as :

(i) Ideal fluids and (ii) Real/actual fluids.

Ideal Fluid

- It is a fluid which does not offer resistance to flow / deformation / change in shape, i.e., it has no viscosity. It is frictionless and incompressible. However, an ideal fluid does not exist in nature and therefore, it is only an imaginary fluid.
- An ideal fluid is the one which offers no resistance to flow/change in shape.

Real Fluid

It is a fluid which offers resistance when it is set in motion. All naturally occurring fluids are real fluids.

7.3.2 Compressible and Incompressible Fluids

Based upon the behaviour of fluids under the action of externally applied pressure and temperature, the fluids are classified as : (i)Compressible fluids (ii) Incompressible fluids.

A fluid possesses a definite density at a given temperature and pressure. Although the density of fluid depends on temperature and pressure, the variation of density with changes in these variables may be large or small.

Compressible Fluid :

- *If the density of a fluid is affected appreciably by changes in temperature and pressure, the fluid* is said to be **compressible.**

If the density of a fluid is sensitive to changes in temperature and pressure, the fluid is said to be **compressible.**

Incompressible Fluid :

- *If the density of a fluid is not appreciably affected by moderate changes in temperature and pressure, the fluid* is said to be **incompressible.**

If the density of a fluid is almost insensitive to moderate changes in temperature and pressure, the fluid is said to be **incompressible.**

Thus, **liquids** are considered to be incompressible fluids, whereas **gases** are considered to be compressible fluids.

7.3.3 Newtonian and Non-Newtonian Fluids

Based upon the behaviour of fluids under the action of shear stress, the fluids are classified as :

(i) Newtonian fluids

(ii) Non-Newtonian fluids.

For most commonly known fluids, a plot of τ versus du/dy results in a straight line passing through the origin and such fluids are called as Newtonian fluids.

Fluids that obey Newton's law of viscosity, i.e., the fluids for which the ratio of the shear stress to the rate of shear or shear rate is constant, are called as **Newtonian fluids.** This is true for all gases and for most pure liquids.

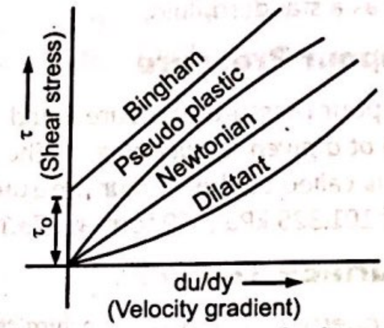


Fig. 7.2 : Shear stress versus velocity gradient for Newtonian and Non-Newtonian fluids

Examples of Newtonian Fluids : All gases, air, liquids, such as kerosene, alcohol, glycerine, benzene, hexane, ether etc., solutions of inorganic salts and of sugar in water.

Fluids for which the ratio of the shear stress to the shear rate is not constant but is considered as a function of rate of shear, i.e., fluids which do not follow Newton's law of viscosity are called as **non-Newtonian fluids**. Generally, liquids particularly those containing a second phase in suspension (solutions of finely divided solids and liquid solutions of large molecular weight materials) are non-Newtonian in behaviour.

Examples of non-Newtonian Fluids : Tooth pastes, paints, gels, jellies, slurries and polymer solutions.

A Newtonian fluid is one that follows Newton's law of viscosity. If viscosity is independent of rate of shear or shear rate, the fluid is said to be Newtonian and if viscosity varies with shear rate, the fluid is said to be non-Newtonian.

7.4 PROPERTIES OF FLUIDS

The properties of fluids are

(i) Mass density (specific mass) or simply density (ρ).

(ii) Weight density (specific weight) (w).

(iii) Vapour pressure.

(iv) Specific gravity.

(v) Viscosity.

(vi) Surface tension and capillarity.

(vii) Compressibility and elasticity.

(viii) Thermal conductivity.

(ix) Specific volume.

7.4.1 Density

Density (ρ) or mass density of a fluid is *the mass of the fluid per unit volume*. In the SI system, it is expressed in kg/m^3. The density of pure water at 277 K (4 °C) is taken as 1000 kg/m^3.

7.4.2 Weight Density

Weight density of a fluid is *the weight of the fluid per unit volume*. In the SI system, it is expressed in N/m^3. Specific weight or weight density of pure water at 277 K (4 °C) is taken as 9810 N/m^3.

The relation between mass density and weight density is

$$w = \rho g$$

where g is the acceleration due to gravity (9.81 m/s^2).

7.4.3 Specific Volume

Specific volume of a fluid is *the volume of the fluid per unit mass*. In the SI system, it is expressed in m^3/kg.

7.4.4 Specific Gravity

The specific gravity of a fluid is *the ratio of the density of the fluid to the density of a standard fluid*. For liquids, water at 277 K (4 °C) is considered/chosen as a standard fluid and for gases, air at NTP (0°C and 760 torr) is considered as a standard fluid.

7.4.5 Vapour Pressure

The vapour pressure of a pure liquid is defined as *the absolute pressure at which the liquid and its vapour are in equilibrium at a given temperature* or The pressure exerted by the vapour (on the surface of a liquid) at equilibrium conditions is called as the vapour pressure of the liquid at a given temperature. Pure air free water exerts a vapour pressure of 101.325 kPa (760 torr) at 373.15 K (100 °C).

7.4.6 Surface Tension

The property of liquid surface film to exert tension is called as the surface tension. It is the force required to maintain a unit length of film in equilibrium. It is denoted by the symbol σ (Greek sigma) and its SI unit is N/m.

7.4.7 Viscosity (Dynamic Viscosity)

- *A fluid undergoes continuous deformation when subjected to a shear stress. The resistance offered by a fluid to its continuous deformation* (when subjected to a shear stress/force) is called viscosity
- The viscosity of a fluid at a given temperature is *a measure of its resistance to flow.*
- The viscosity of a fluid (gas or liquid) is practically independent of the pressure for the range that is normally encountered in practice. However, it varies with temperature. For gases, viscosity increases with an increase in temperature, while for liquids it decreases with an increase in temperature. It is denoted by the symbol μ (Greek mu). In the C.G.S. system, viscosity may be expressed in poise (P) or centipoise (cP).

The unit of viscosity in the SI system is Pa·s (kg/cm.s).

$$1 \text{ poise} = 1 \text{ P} = 100 \text{ cP} = 0.10 \text{ kg/(m.s)} = 0.10 \text{ (N.s)/m}^2 \text{ or Pa.s}$$

7.4.8 Kinematic Viscosity

The kinematic viscosity of a fluid is the ratio of the viscosity to the density of the fluid. It is denoted by the symbol υ (Greek Nu) and has the units of m^2/s. ... (7.3)

$$\upsilon = \mu/\rho$$

In units, $$m^2/s = \frac{kg/(m.s)}{kg/m^3} \Rightarrow m^2/s$$

7.5 PRESSURE

The basic property of a static fluid is pressure. When a certain mass of fluid is contained in a vessel, it exerts forces at all points on the surfaces of the vessel in contact. The forces so exerted always act in the direction normal to the surface in contact. *The normal force exerted by a fluid per unit area of the surface* is called as the **fluid pressure**. If F is the force acting on the area A, then the pressure or intensity of pressure is given by

$$P = F/A \qquad \dots (7.4)$$

In a static fluid, the pressure at any given point is the same in all the directions. If the pressure at a given point was not the same in all directions, there would be non-equilibrium and the resultant force should exist. As the fluid is in static equilibrium, there is no net unbalanced force at any point. Hence, the pressure in all directions is the same and thus independent of direction.

In the SI system, the unit N/m^2 is called the pascal, symbol Pa (in honour of the scientist Pascal). As the unit of pressure Pa is small in magnitude, the pressure is usually expressed in SI in kilopascal (kPa). A multiple of pascal is called the bar and it is also used as a unit of pressure.

$$1 \text{ bar} = 10^5 \text{ Pa} = 10^5 \text{ N/m}^2$$

7.5.1 Atmospheric Pressure

The air of the atmosphere exerts pressure on all bodies that are exposed to it. The pressure exerted by the air of the atmosphere is called the atmospheric pressure. At the mean sea level, the pressure exerted by the air is 101325 N/m^2 (101325 Pa = 101.325 kPa). This pressure is known as the standard or normal atmospheric pressure or simply atmospheric pressure. One standard atmosphere (1 atm) balances a column of 760 mm of mercury at 0°C.

$$\begin{aligned} 1 \text{ atm} &= 760 \text{ mm Hg} = 760 \text{ Torr} \\ &= 101325 \text{ Pa or N/m}^2 \\ &= 101.325 \text{ kPa or kN/m}^2 \\ &= 1.103 \text{ bar} = 760 \text{ torr} \\ &= 10.33 \text{ m } H_2O \\ &= 1.033 \text{ kgf/cm}^2 \\ &= 14.7 \text{ psi} \end{aligned}$$

7.5.2 Vacuum (Vacuum Pressure)

Vacuum refers to sub-atmospheric pressure, i.e., pressure below the normal atmospheric pressure. It is usually expressed in the SI system in Torr (in honour of the scientist Torricelli). This unit is used for systems operating under vacuum. The pressure exerted by 1 millimetre column of mercury is named as Torr.

$$1 \text{ Torr} = 1 \text{ mmHg}$$

7.5.3 Absolute Pressure

It is the pressure prevailing in a vessel/pipe line/process equipment. It is the true/actual pressure or pressure above reference zero.

It is the pressure measured from absolute zero.

The absolute pressure at a point in a fluid is equal to the sum of the gauge and the atmospheric pressures.

7.5.4 Gauge Pressure

Normally, the pressure is measured with the help of a pressure gauge. The pressure gauge measures the pressure above the atmospheric pressure. Thus, the atmospheric pressure on the gauge scale is marked as zero (i.e., the atmospheric pressure is taken as datum). The pressure registered by the pressure gauge is called the gauge pressure and therefore the letter 'g' follows the unit of pressure [1.2 (kgf/cm^2) g, 2 atmg, 300 kPag, 30 psig].

The gauge pressure does not indicate the true total pressure (absolute pressure). For obtaining the prevailing true pressure or pressure above reference zero, we have to add the atmospheric or barometric pressure in consistent units to the gauge pressure. If no letter follows the pressure units, it is taken as absolute pressure.

The relationship between absolute and gauge pressure is

Absolute pressure = gauge pressure + atmospheric pressure

Vacuum refers to sub-atmospheric pressure (pressure below the atmospheric pressure).

The relationship between absolute pressure and vacuum is

Absolute pressure = atmospheric pressure – vacuum

The pressure measuring devices are of two types: tube gauges and mechanical gauges.

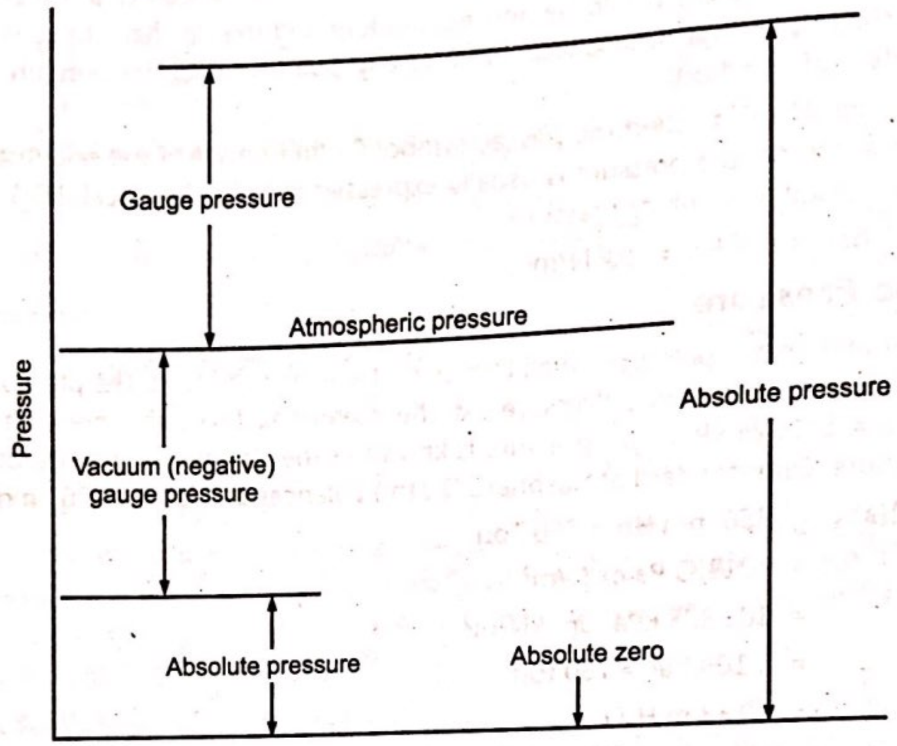


Fig. 7.3 : Pressure relationships

7.6 NEWTON'S LAW OF VISCOSITY

It states that the *shear stress on a layer of a fluid is directly proportional to the rate of shear.* Mathematically,

$$\tau \propto \frac{u}{y}, \text{ i.e., } \tau = \mu \cdot \frac{u}{y}$$

Consider two layers of a fluid 'y' cm apart as shown in Fig. 7.4. Let the area of each of these layers be A cm^2. Assume that the top layer is moving parallel to the bottom layer at a velocity of 'u' cm/s relative to the bottom layer. To maintain this motion, i.e., the velocity 'u' and to overcome the fluid friction between these layers, for any actual fluid, a force of 'F' dyne (dyn) is required.

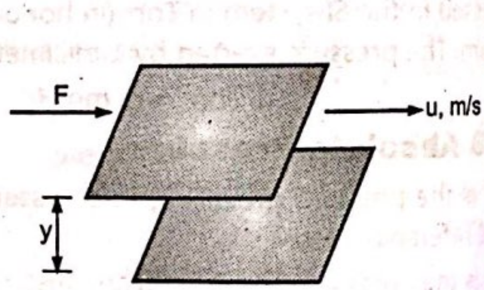


Fig. 7.4 : Definition of viscosity

Experimentally it has been found that the force F is directly proportional to the velocity u and area A, and inversely proportional to the distance y.

Therefore, mathematically it becomes

$$F \propto u.A/y \quad \text{... (7.5)}$$

Introducing a proportionality constant μ (Greek 'mu'), Equation (7.5) becomes

$$F = \mu\, u\, A/y$$

$$F/A = \mu.u/y \quad \text{... (7.6)}$$

Shear stress, τ (Greek 'tau') equal to F/A between any two layers of a fluid may be expressed as

$$\tau = F/A = \mu.u/y \quad ... (7.7)$$

The above equation in a differential form becomes

$$\tau = \mu . \frac{du}{dy} \quad ... (7.8)$$

(The ratio u/y can be replaced by the velocity gradient du/dy.)

In the SI system, the shear stress τ is expressed in N/m^2 and the velocity gradient/shear rate or rate of shear deformation is expressed in 1/s or s^{-1}.

Equation (7.8) is called Newton's law of viscosity. In the rearranged form, it serves to define the proportionality constant as

$$\mu = \frac{\tau}{du/dy} \quad ... (7.9)$$

which is called as the coefficient of viscosity, or the dynamic viscosity (since it involves force), or simply the viscosity of a fluid. Hence, the dynamic viscosity μ, may be defined as the shear *stress required to produce unit rate of shear deformation (or shear rate).*

Viscosity is the property of a fluid and in the SI system it has the units of $(N.s)/m^2$ or Pa.s or kg/(m.s).

As the unit $(N.s)/m^2$ is very large for most of the fluids, it is customary to express viscosity as $(mN.s)/m^2$ or mPa.s, where mN is millinewtons, i.e., 10^{-3} N and mPa is millipascal, i.e., 10^{-3} Pa.

In the C.G.S. system, viscosity may be expressed in poise (P) (the unit poise is named after the French scientist Poiseuille) or centipoise (cP).

$$1 \text{ poise} = 1 \text{ P} = 1 \text{ gm/(cm.s)} = 0.10 \text{ kg/(m.s)} = 0.10 \text{ (N.s)/m}^2 \text{ or Pa.s} = 100 \text{ cP}$$

In many problems involving viscosity, there appears a term kinematic viscosity.

The kinematic viscosity of a fluid is defined as *the ratio of the viscosity of the fluid to its density* and is denoted by the symbol υ (Greek 'nu').

$$\upsilon = \mu/\rho$$

In the SI system, υ has the units of m^2/s. The C.G.S. unit of kinematic viscosity is termed as stoke and is equal to cm^2/s.

7.7 HYDROSTATIC EQUILIBRIUM - CONCEPT AND APPLICATIONS

- The term hydrostatics means the study of pressure exerted by a liquid or a gas at rest. The direction of such a pressure is always perpendicular to the surface on which it acts.
- In fluid mechanics, a fluid is said to be in hydrostatic equilibrium or hydrostatic balance when it is at rest.
- A system is in hydrostatic equilibrium when the force due to pressure exactly balances the force due to gravity.
- The principle of hydrostatic equilibrium is that the pressure at any point within a fluid at rest (when hydrostatic) is only due to the weight of the overlying fluid.
- The pressure at the bottom of fluid in a vertical tank is the weight of a column of the fluid on unit cross-section area of the tank.
- Some applications of hydrostatic equilibrium are :

  (i) Barometric equation.

  (ii) Manometers.

  (iii) Decanters.

  (iv) Centrifuge.

- A U-tube manometer is the simplest form of manometer. It consists of a small **diameter** U-shaped glass tube. The tube is clamped on a wooden board. A scale is fixed on the same board between the two arms/limbs/legs of the manometer. The U-tube is partially filled with a manometric fluid which is heavier than the process fluid. The manometric fluid is immiscible with the process fluid. The common manometric fluid is **mercury**.

Pressure Measurement by U-tube Manometer

- A manometer works on the principle of hydrostatic equilibrium.
- A pressure measuring device using liquid columns in vertical or inclined tube is called a manometer.
- Manometers are called tube gauges to measure the pressure (P) or differential pressure (ΔP).

U-tube manometers are used to measure the pressure or differential pressure. When U-tube manometer is used to measure the pressure of a fluid flowing through a pipeline, one arm of it is connected to the pipeline (gauge point) and the other arm is open to the atmosphere, in which the manometric fluid can rise freely.

- U-tube manometer is filled with a given manometric fluid (fluid M) upto a certain height. The remaining portion of the U-tube is filled with the process fluid/flowing fluid of density ρ including the tubings. One limb of the manometer is connected to the upstream tap in a pipeline and the other limb is connected to the downstream tap in the pipeline between which the pressure difference $P_1 - P_2$ is required to be measured. Air, if any, is there in the line connecting taps and manometer is removed. At steady state, for a given flow rate, the reading of the manometer, i.e., the difference in the level of the manometric fluid in the two arms is measured and it gives the value of pressure difference in terms of manometric fluid across the taps (stations). It may then be converted in terms of m of flowing fluid.
- Consider a U-tube manometer as shown in Fig. 7.4 connected in a pipeline. Let pressure P_1 be exerted in one limb of the manometer and pressure P_2 be exerted in the another limb of the manometer. If P_1 is greater than P_2, the interface between the two liquids in the limb 1 will be depressed by a distance 'h' (say) below that in the limb 2. To arrive at a relationship between the pressure difference $(P_1 - P_2)$ and the difference in the level in the two limbs of the manometer in terms of manometric fluid (h), pressures at points 1, 2, 3, 4 and 5 are considered.

Pressure at point 1 = P_1

Pressure at point 2 = $P_1 + (x + h)\,\rho.g$

Pressure at point 3 = Pressure at point 2 = $P_1 + (x + h)\,\rho.g$

(as the points 2 and 3 are at the same horizontal plane).

Pressure at point 4 = $P_1 + (x + h)\,\rho.g - h\,.\,\rho_M.g$

Pressure at point 5 = $P_1 + (x + h)\,\rho.g - h\,.\,\rho_M.g - x.\rho.g$

Pressure at point 5 = P_2

Then, we can write,

$$P_2 = P_1 + (x + h)\,.\,\rho.g - h\,\rho_M.g - x\,.\,\rho.g \quad \text{... (7.24)}$$

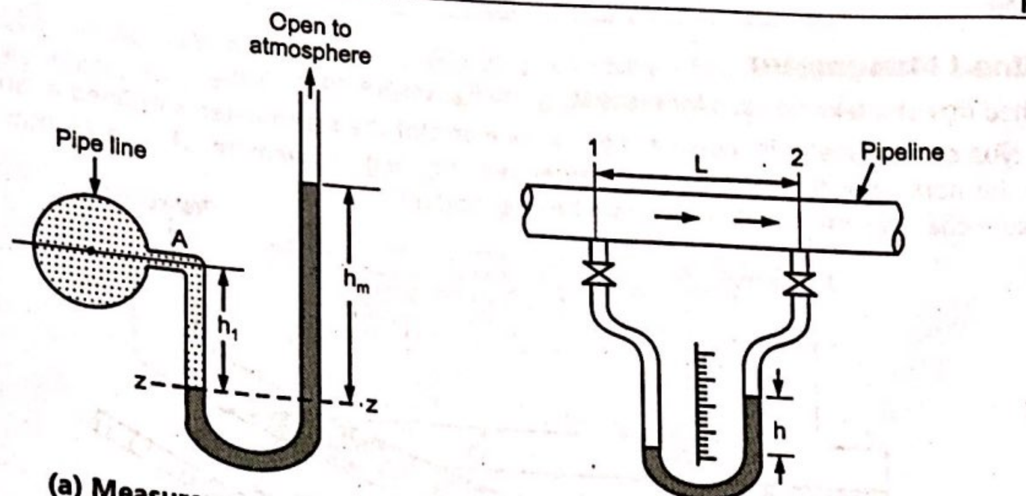
$$P_1 - P_2 = \Delta P = h\,(\rho_M - \rho)g \quad \text{... (7.25)}$$

where ΔP is the pressure difference and 'h' is the difference in levels in the two arms of the manometer in terms of manometric fluid.

If the flowing fluid is a gas, density ρ of the gas will normally be small compared with the density of the manometric fluid, ρ_M and thus Equation (7.25) reduces to

$$\Delta P = P_1 - P_2 = h\,.\,\rho_M.g \quad \text{... (7.26)}$$

When U-tube manometer is used to measure the differential pressure, the arms of the manometer are connected to different locations of a pipe line along the length of the pipeline over which we have to measure the differential pressure. Note that the manometric fluid should be heavier than the flowing fluid and it should not mix with the flowing fluid.



(a) Measurement of pressure **(b) Measurement of differential pressure**

Fig. 7.7 : Pressure measurement by U-tube manometer

The open U-tube manometer [Fig. 7.7] shows a reading h_m of manometric fluid. The interface between the manometric fluid and the fluid of which the pressure is to be measured is h_1 meters below the point of attachment A.

Let ρ be the density of the flowing fluid and ρ_m be the density of the manometric fluid.

Then, gauge pressure P_A at A is

$$P_A = h_m \rho_m g - h_1 \rho g \quad \ldots (7.27)$$

Let us obtain the above equation.

Let

z-z = the datum line as shown in Fig. 7.7.

h_m = height of the manometric fluid above the datum line.

h_1 = height of the flowing fluid above the datum line.

Pressure in the left arm/limb above the datum line = $P_A + h_1 \rho g$, N/m^2.

Pressure in the right arm/limb above the datum line = $h_m \rho_m g$, N/m^2.

Since the pressure in both the arms above the z-z datum is equal, we can write

$$P_A + h_1 \rho g = h_m \rho_m g$$

$$P_A = h_m \rho_m g - h_1 \rho g$$

Th head in meters of the flowing fluid is given by

$$h_A = \frac{P_A}{\rho g} = \frac{h_m \rho_m g - h_1 \rho g}{\rho g}$$

$$h_A = h_m \left(\frac{\rho_m}{\rho g}\right) - h_1 \text{ ... m of flowing fluid} \quad \ldots (7.28)$$

The differential pressure or pressure difference between two points within a fluid across which a U-tube manometer is connected is given by the relation :

$$(P_1 - P_2) = \Delta P = h\,(\rho_m - \rho)\,g \quad \ldots (7.29)$$

where h is the difference in levels in the two arms of the U-tube manometer in terms of the manometric fluid [difference in levels of the manometric fluid in the two arms of the manometer].

Here, $P_1 > P_2$. P_1 and P_2 are expressed in N/m^2, ρ in kg/m^3, h in m and g in m/s^2.

If the flowing fluid is a gas, then the density, ρ, of the gas is normally small compared to the density of the manometric fluid. Thus, it can be neglected and with this Equation (7.29) reduces to :

$$P_1 - P_2 = \Delta P = h\,\rho_m\,g \quad \ldots (7.30)$$

7.9.2 Inclined Manometer

- Inclined manometers are used for measuring small pressure differences.
- This type of manometer is shown in Fig. 7.8. One arm of the manometer is inclined at an angle of 5 to 10° with the horizontal so as to obtain a larger reading. (e.g., movement of 7 to 10 mm is obtained for a pressure change corresponding to 1 mm head of liquid.)

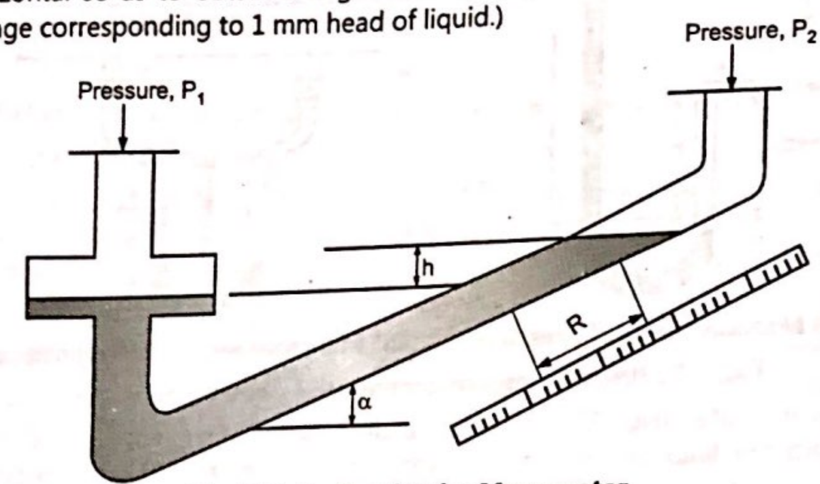


Fig. 7.8 : Inclined Tube Manometer

- In the vertical leg of this manometer an enlargement is provided so that the movement of the meniscus in this enlargement is negligible within the operating range of the manometer. If the reading R(in m) is taken as shown, i.e., distance travelled by the meniscus of the manometric fluid along the tube, then

$$h = R \sin \alpha \quad \text{... (7.31)}$$

where α = angle of inclination

and $$(P_1 - P_2) = R \sin \alpha \, (\rho_M - \rho) \, g \quad \text{... (7.32)}$$

7.9.3 Differential Manometer / Two Liquid Manometer / Multiplying Gauge

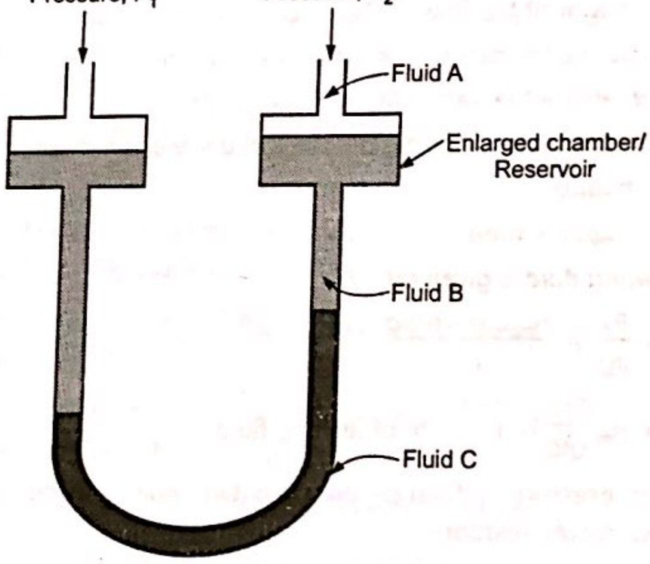


Fig. 7.9 (A) : Differential Manometer

- Differential manometer is used for the measurement of very small pressure differences or for the measurement of pressure differences with a very high precision. It may often be used for gases.
- It consists of a U-tube made of glass. The ends of the tube are connected to two enlarged transparent chambers / reservoirs. The reservoirs at the ends of each arm are of a large cross-section than that of the tube. The manometer contains two manometric liquids of different densities and these are immiscible with each other and with the fluid for which the pressure difference is to be measured. This type of manometer is shown in Fig. 7.9 (A).

- The densities of the manometric fluids are nearly equal to have a high sensitivity of the manometer. Liquids which give sharp interfaces are commonly used, e.g., paraffin oil and industrial alcohol, etc.

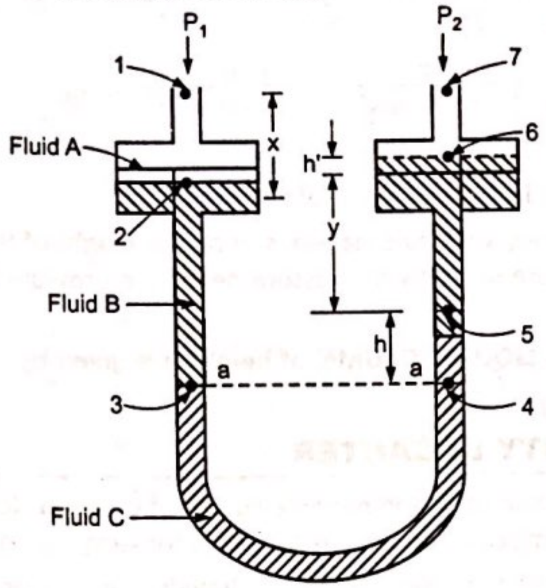


Fig. 7.9 (B) : Differential Manometer (for pressure balance)

Let the flowing fluid be 'A' of density ρ_A and manometric fluids be B and C of densities ρ_B and ρ_C ($\rho_C > \rho_B$), respectively [$\rho_A < \rho_B$ and ρ_C].

The pressure difference between two points (1 and 7) can be obtained by writing down pressures at points 1, 2, 3, 4, 5, 6, and 7 and is given by

$$P_1 - P_2 = h'(\rho_B - \rho_A)\,g + h(\rho_C - \rho_B)\,g \qquad \text{... (7.33)}$$

If the level of liquid in two reservoirs is approximately same, then $h' \approx 0$ and Equation (7.33) reduces to

$$P_1 - P_2 = h(\rho_C - \rho_B)\,g \qquad \text{... (7.34)}$$

where h is the difference in level in the two arms/limbs of the manometer.

When the densities ρ_B and ρ_C are nearly equal [($\rho_C - \rho_B$) small], then very large values of h can be obtained for small pressure differences.

Alternately, the pressure at the level a – a in Fig. 7.7 must be the same in each of the limbs and therefore,

$$P_1 + [x.\rho_A + h'\,\rho_A + y.\rho_B + h.\rho_B]\,g = P_2 + [x.\rho_A + h'\,\rho_B + y\,.\rho_B + h.\rho_C]\,g \qquad \text{... (7.35)}$$

$$\therefore \quad (P_1 - P_2) = h'(\rho_B - \rho_A)\,g + h(\rho_C - \rho_B)\,g \qquad \text{... (7.36)}$$

7.10 PRESSURE EXERTED BY A LIQUID COLUMN

Pressure : The basic property of a static fluid is pressure. When a certain mass of fluid is contained in a vessel, it exerts forces at all points on the surfaces of the vessel in contact. The forces so exerted always act in the direction normal to the surface in contact. *The normal force exerted by a fluid per unit area of the surface* is called as the **fluid pressure**. If F is the force acting on the area A, then the pressure or intensity of pressure is given by

$$P = F/A \qquad \text{... (7.37)}$$

In a static fluid, the pressure at any given point is the same in all the directions. If the pressure at a given point was not the same in all directions, there would be non-equilibrium and the resultant force should exist. As the fluid is in static equilibrium, there is no net unbalanced force at any point. Hence, the pressure in all directions is the same and thus independent of direction.

7.17 POINT VELOCITY

It is the velocity of a flowing fluid at a certain location within the stream of the fluid.

7.18 TYPES OF FLOW

7.18.1 Steady Flow

- The flow is said to be steady if it does not vary with time.
- If the mass flow rate and/or the quantities such as temperature and pressure at a point in the flow system do not change/vary with time, then the flow is said to be steady.
- A flow which is invariant with time is steady.

7.18.2 Unsteady Flow

- The flow is said to be unsteady if it varies with time.
- A flow which is time dependent is unsteady.
- If the mass flow rate and/or the quantities such as temperature and pressure at a point in the flow system vary/change with time, then the flow is said to be unsteady.

7.18.3 Potential Flow

The flow of incompressible fluids without the presence of shear is known as potential flow. In potential flow, eddies and cross currents cannot form within the stream and friction cannot develop.

7.18.4 Fully Developed Flow

The flow with unchanging velocity distribution is called fully developed flow.

7.18.5 Compressible Flow

The flow in which the volume of a flowing fluid and thus its density changes during the flow is called compressible flow.

7.18.6 Incompressible Flow

The flow in which the volume of a flowing fluid and thus its density does not change during the flow is called. incompressible flow.

7.19 STREAM LINE AND STREAM TUBE

Stream line :

A stream line is an imaginary line in the fluid such that the fluid velocity at any point is tangential to it. As the velocity is tangent to the streamline, no matter can cross it.

Stream tube :

A stream tube is an element of fluid bounded by a number of streamlines. The stream tube may be of any convenient cross-sectional shape and no net flow occurs across it.

7.20 EQUATION OF CONTINUITY OF FLUID FLOW

- When a fluid is in motion, it must move in such a way that its mass is conserved along a flow path.
- If a fluid is continuously flowing through a conduit (pipe or channel) of constant or varying cross-sectional area, the quantity of the fluid passing per second on weight basis is the same at all locations of the conduit.

  Mathematically, $\dot{m} = \rho u A = \text{constant}$

This is known as the equation of continuity. It is the first and fundamental equation of fluid flow.

The equation of continuity is a mathematical expression for the law of conservation of mass. According to the law of conservation of mass for a steady flow system, the rate of mass entering the flow system is equal to that leaving as accumulation is either constant or nil in the flow system under steady conditions.

[In a steady state flow system the values of the quantity of a fluid flowing through the system and variables of the system do not change with time.

Consider a flow system (a stream tube of varying cross-section) as shown in Fig. 7.11.

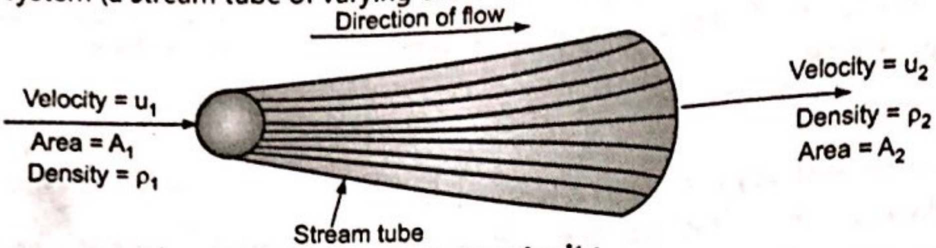


Fig. 7.11 : Continuity

As the flow cannot take place across the walls of the stream tube, the rate of mass entering the stream tube must be equal to that leaving. Let u_1, ρ_1 and A_1 be the average velocity of the fluid, the density of the fluid and cross-section area of the tube at the entrance, and let u_2, ρ_2 and A_2 be the corresponding quantities at the exit of the tube. Assume that the flow to be potential flow and the density to be constant in a single cross-section.

Rate of mass entering the flow system = $\rho_1 u_1 A_1$

Rate of mass leaving the flow system = $\rho_2 u_2 A_2$

Let $\dot{m}$ be the rate of flow in mass per unit time (mass flow rate of the flowing fluid).

Under steady flow conditions, according to the law of conservation of mass, the mass of fluid entering the tube in unit time is the same as that leaving the tube. Therefore,

$$\dot{m} = \rho_1 u_1 A_1 = \rho_2 u_2 A_2 \quad \text{... (7.54)}$$

From Equation (7.54), it follows for a stream tube or a pipe/channel of varying/constant cross-sectional area

$$\dot{m} = \rho\, uA = \text{constant}$$

OR :

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2 \quad \text{... (7.55)}$$

Equation (7.55) is known as the *equation of continuity*. It is applicable to compressible as well as to incompressible fluids. In the case of incompressible fluids, $\rho_1 = \rho_2 = \rho$.

The equation of continuity is useful for calculating the velocity of a fluid flowing through pipes of different diameters.

Assume that we know the velocity (u_1) of a fluid through a pipe of diameter D_1 and we have to obtain the velocity of the fluid (u_2) through a pipe of diameter D_2 which is connected to the pipe of diameter D_1. Then, from the equation of continuity,

$$\dot{m} = \rho_1 u_1 A_1 = \rho_2 u_2 A_2 \quad \text{... (7.56)}$$

but $\rho_1 = \rho_2$

$$\therefore \quad u_1 A_1 = u_2 A_2 \quad \text{... (7.57)}$$

where A_1 = cross-section area of the pipe of diameter $D_1 = \pi/4\, D_1^2$

A_2 = cross-section area of the pipe of diameter $D_2 = \pi/4\, D_2^2$

From Equation (7.57), it is clear that $u \propto 1/A$. That is the velocity of a fluid in a steady flow at any point is inversely proportional to the cross-sectional area of the pipe at that point.

Substituting for A_1 and A_2, Equation (7.57) becomes

$$u_1 (\pi/4.D_1^2) = u_2 (\pi/4.D_2^2)$$

$$u_2 = u_1 (D_1^2 / D_2^2) \quad \text{... (7.58)}$$

If $D_2 > D_1$, then $u_2 < u_1$

and if $D_2 < D_1$, then $u_2 > u_1$

When A is expressed in m^2, density in kg/m^3 and velocity in m/s, the unit of mass flow rate is kg/s.

We have : $\dot{m} = \rho\, u\, A$

Unit of $\dot{m} \Rightarrow (kg/m^3) \times (m/s) \times (m^2)$

$\Rightarrow kg/s$

7.24 CONCEPT OF CRITICAL VELOCITY

The velocity at which the flow changes from laminar to turbulent is known as the critical velocity. The value of critical velocity corresponds to N_{Re} = 2100. It is obtained as follows

$$N_{Rec} = \frac{D u_c \rho}{\mu} = 2100$$

$$\therefore \quad u_c = 2100 \times \mu / D \cdot \rho \quad \text{... (7.61)}$$

where u_c is the critical velocity.

7.25 DIFFERENT TYPES OF ENERGIES ASSOCIATED WITH FLOWING FLUIDS

7.25.1 Pressure Energy

It is the work which must be done in order to introduce a fluid into a system without change in the volume. It is the energy of the fluid due to pressure acting on it.

$$\begin{aligned} \text{Pressure energy (flow energy)} &= \text{work done on a fluid} \\ &= \text{force} \times \text{displacement} \\ &= \frac{\text{force}}{\text{area}} \times \text{area} \times \text{displacement} \\ &= \text{pressure} \times \text{volume} \end{aligned}$$

$$\begin{aligned} \text{Pressure energy per unit mass of fluid} &= \frac{\text{pressure} \times \text{volume}}{\text{mass}} = \frac{\text{pressure}}{\text{mass/volume}} \\ &= \frac{P}{\rho}, \text{ J/kg in the SI system.} \end{aligned}$$

7.25.2 Kinetic Energy

It is the energy of a fluid by virtue of its motion with reference to some arbitrarily fixed body.

The kinetic energy of a fluid of mass m moving with velocity u is given by

$$\text{Kinetic energy} = \frac{1}{2} m u^2$$

$$\text{Kinetic energy per unit mass of fluid} = \frac{1}{2} u^2, \text{ J/kg in the S.I. system}$$

7.25.3 Potential Energy

It is the energy of a fluid due to its position in the earth's gravitational field. It is equal to the work that must be done on the fluid in order to raise it to a certain position from some arbitrarily chosen datum level.

At a datum level, the potential energy is taken as zero.

The potential energy of a fluid of mass m situated at a height Z above a datum level is given by

$$\text{Potential energy} = mgZ$$

$$\text{Potential energy per unit mass} = g\,Z, \text{ J/kg in the S.I. system}$$

7.26 BERNOULLI'S EQUATION

7.26.1 Bernoulli's Equation (Theorem) Statement

- Bernoulli's equation states that for an incompressible, steady and inviscid fluid flow, the total mechanical energy of the fluid is constant.
- Bernoulli equation states that for a incompressible fluid flowing steadily through a conduit, the total mechanical energy of the fluid (comprising the energy associated with fluid pressure, the gravitational potential energy and the kinetic energy of fluid motion) remains the same (or remains constant), while the fluid moves from one location to another in the conduit.

The mathematical statement of Bernoulli's equation is

$$\frac{P}{\rho} + gZ + \frac{u^2}{2} = \text{constant} \quad \text{... (7.62)}$$

The unit of each term is J/kg in the SI system, so each term represents energy per unit mass of fluid.

OR :

$$\frac{P}{\rho g} + Z + \frac{u^2}{2g} = \text{constant} \quad \text{... (7.63)}$$

7.26.2 Assumptions in Deriving Bernoulli's Equation

The following assumptions are made in deriving Bernoulli's equation.

(i) The flow is steady [i.e., the properties of a fluid (e.g., density, velocity) at any particular point or location in the flowing fluid do not change with time].

(ii) The flow is incompressible [i.e., eventhough the pressure varies, the density does not vary along a streamline (i.e., it remains constant)].

(iii) The fluid is flowing along the streamline.

(iv) The flow is irrotational (potential flow).

(v) The fluid is inviscid [for a inviscid fluid, the viscosity is zero and hence the flow occurs without friction, i.e., the flow is frictionless. There is no energy dissipation.]

[An inviscid flow is a flow of an ideal fluid which is assumed to have no viscosity.]

7.26.3 Bernoulli Equation Derivation

An important relation, called the Bernoulli equation without friction can be derived on the basis of Newton's second law of motion (force is equal to the rate of change of momentum) for potential flow. It is simply an energy balance. The variation of velocity across a given cross-section, effect of frictional forces are neglected at first and corrections for the same are then made in the equation. Thus, the relation that will be obtained is strictly applicable to an inviscid (frictionless) fluid.

Let us consider an element of length ΔL of a stream tube of constant cross-sectional area as shown in Fig. 7.14.

Let us assume that the cross-sectional area of element be A and the density of the fluid be ρ. Let u and P be the velocity and pressure at the entrance (upstream), and u + Δu, P + ΔP are the corresponding quantities at the exit (downstream).

The forces acting on the element (treating the element as a free body) are

1. The force from the upstream pressure = PA (i.e., the force acting in the direction of flow, taken as positive)
2. The force from the downstream pressure normal to the cross-section of the tube = (P + ΔP) A (i.e., the force opposing the flow, taken as negative)
3. The force from the weight of fluid [i.e., the force of gravity acting downward (taken as negative)] = ρA ΔL.g

The component of this force acting opposite to the direction of flow is ρ AΔL g cos θ.

Of the three forces cited above, the first one helps the flow while the remaining two forces oppose the flow.

Rate of change of momentum of the fluid along the fluid element = $\dot{m}\,[u + \Delta u - u] = \dot{m}\,\Delta u = \rho\, u\, A\, \Delta u$

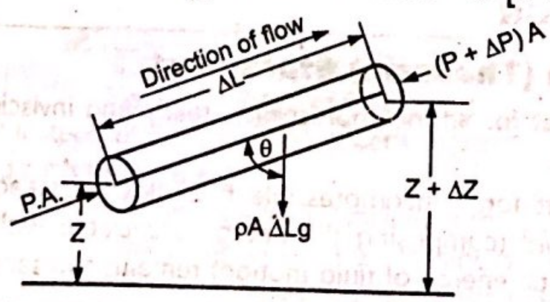


Fig. 7.14 : Force balance for potential flow

According to the Newton's second law of motion,

$$\left\{\begin{matrix}\text{Sum of all forces} \\ \text{acting in the direction of flow}\end{matrix}\right\} = \left\{\begin{matrix}\text{Rate of change of} \\ \text{momentum of a fluid}\end{matrix}\right\} \quad \text{... (7.64)}$$

$$PA - (P + \Delta P)\,A - \rho\,A\,\Delta L\,g\cos\theta = \rho\,.\,u.A\,\Delta u$$

$$-\Delta P.A - \rho\,A\,\Delta L\,g\cos\theta = \rho\,.\,u.A\,\Delta u$$

$$\therefore \quad \Delta P.A + \rho A\,\Delta L\,g\cos\theta + \rho.u.A\,\Delta u = 0 \quad \text{... (7.65)}$$

Dividing each term of Equation (7.65) by A.ΔL.ρ, we get

$$\frac{\Delta P}{\rho \Delta L} + g\cos\theta + \frac{u.\Delta u}{\Delta L} = 0 \quad \text{... (7.66)}$$

But

$$\cos\theta = \Delta Z/\Delta L$$

Substituting the value of cos θ in Equation (7.66) gives

$$\frac{1}{\rho}\frac{\Delta P}{\Delta L} + g\frac{\Delta Z}{\Delta L} + u\frac{\Delta u}{\Delta L} = 0 \quad \text{... (7.67)}$$

If we express the changes in the pressure, velocity, height, etc. in differential form, then Equation (7.67) becomes

$$\frac{1}{\rho}\frac{dP}{dL} + g\frac{dZ}{dL} + \frac{d\,(u^2/2)}{dL}$$

which can be rewritten as :

$$\frac{dP}{\rho} + g\,dZ + d\,(u^2/2) = 0 \quad \text{... (7.68)}$$

Equation (7.68) is known as the *Bernoulli equation*. It is the differential form of the Bernoulli equation. For incompressible fluids, density is independent of pressure and hence, the integrated form of Equation (7.68) is

$$\frac{P}{\rho} + gZ + \frac{u^2}{2} = \text{constant} \quad \text{... (7.69)}$$

Thus, the Bernoulli equation, Equation (7.69), relates the pressure at a point in the fluid to its position and velocity, i.e., pressure energy at a point within a fluid to its potential energy and kinetic energy.

$$\frac{P}{\rho} = \text{pressure energy}$$

$$gZ = \text{potential energy}$$

and

$$\frac{u^2}{2} = \text{kinetic energy}$$

Each term in the Bernoulli equation [Equation (7.69)] represents energy per unit mass of the fluid and has the units of J/kg in the SI system.

Let us check the unit of each term.

The unit of $\frac{P}{\rho}$ is : $\left(\frac{N}{m^2}\right) \times \frac{1}{(kg/m^3)} = \frac{N.m}{kg} = J/kg$

The unit of g Z is : $\left(\frac{m}{s^2}\right)(m) = \left(\frac{kg}{kg}\right)\left(\frac{m}{s^2}\right)(m) = \left(\frac{kg.m}{s^2}\right)\left(\frac{m}{kg}\right)$

$$= \frac{N.m}{kg} = J/kg$$

The unit of $u^2/2$ is : $\frac{m^2}{s^2} = \left(\frac{kg}{kg}\right)\left(\frac{m^2}{s^2}\right) = \left(\frac{kg.m}{s^2}\right)\left(\frac{m}{kg}\right) = \frac{N.m}{kg} = J/kg$

[As 1 N = 1 (kg.m)/s²]

Equation (7.69) can be written in an alternate form as :

$$\frac{P}{\rho g} + Z + \frac{u^2}{2g} = \text{constant} \quad \text{... (7.70)}$$

Equation (7.70) is obtained by dividing each term in Equation (7.69) by g.

Equation (7.70) is the alternate form of the Bernoulli equation and each term in this equation represents energy per unit weight of the fluid and has the dimensions of length. Hence, each term of Equation (7.70) is regarded as the head that is contributing to the *total fluid head.*

The unit of $\frac{P}{\rho g}$ is $\left(\frac{N}{m^2}\right)\left(\frac{1}{\frac{kg}{m^3}}\right)\left(\frac{1}{m/s^2}\right) \Rightarrow \frac{N.s^2}{kg} \Rightarrow \frac{kg.m}{s^2} \times \frac{s^2}{kg} = m$

The unit of $u^2/2g$ is $(m/s)^2 \times \frac{1}{(m/s^2)} \Rightarrow m$

Therefore, $\frac{P}{\rho g}$ is the pressure head / static head, Z is the potential head and $u^2/2g$ is the velocity head/kinetic head.

Weight density (w) = mass density (ρ) × g

Kinetic energy per unit weight = kinetic head or velocity head

$$= \frac{1/2 \,.\, mu^2}{mg} = u^2/2g \text{ , m}$$

Potential energy per unit weight = Potential head

$$= \frac{mg\,Z}{mg} = Z \text{ , m}$$

Pressure energy per unit weight = Pressure head

$$= \frac{\text{pressure} \times \text{volume}}{\text{weight}} = \frac{\text{pressure}}{\text{weight/volume}} = \frac{P}{\text{weight density}}$$

$$= \frac{P}{\rho g} \text{, m}$$

The sum of the pressure head, velocity head, and potential head is known as the total head or the total energy per unit weight of the fluid. The Bernoulli equation states that *in a steady irrotational flow of an incompressible fluid, the total energy at any point is constant.*

The sum of the pressure head and potential head, i.e., $P/\rho g + Z$ is termed as the piezometric head.

Equation (7.69) when applied between stations 1 and 2 in the direction of flow becomes

$$\frac{P_1}{\rho} + g\,Z_1 + \frac{u_1^2}{2} = \frac{P_2}{\rho} + g\,Z_2 + \frac{u_2^2}{2} \quad \text{... (7.71)}$$

7.27 CORRECTIONS FOR BERNOULLI'S EQUATION

7.27.1 Kinetic Energy Correction

In the previous discussion, it is assumed that the velocity u to be constant over the area A. But in actual practice, the velocity varies over a single cross section and we have a velocity profile over the cross section. The velocity of the fluid is zero at the wall surface and maximum at the centre of the pipe. Hence, allowance must be made for the velocity profile in the kinetic energy term. This can be done by introducing a correction factor α into the kinetic energy term. The kinetic energy term would be written as $\frac{\alpha u^2}{2}$. For the flow of a fluid through a circular cross-section, α = 2 for laminar flow and α = 1 for turbulent flow.

Since η is always less than one, the mechanical energy delivered to the fluid (ηW_p) is less than the work done by the pump. The Bernoulli equation corrected for the pump work between stations 1 and 2 is thus given by

$$\frac{P_1}{\rho} + gZ_1 + \frac{\alpha_1 u_1^2}{2} + \eta W_p = \frac{P_2}{\rho} + g Z_2 + \frac{\alpha_2 u_2^2}{2} + h_f \quad \text{... (7.75)}$$

7.29 BERNOULLI'S EQUATION : SIGNIFICANCE AND APPLICATIONS

Whenever the fluid is inviscid (non-viscous) and incompressible and the flow of fluid is steady, the flow can be taken into consideration from an energy perspective.

In fluid dynamics, Bernoulli's equation is a relation among the pressure, velocity and elevation in a flowing fluid, the compressibility and viscosity of the fluid are negligible and the flow is steady.

Therefore, Bernoulli's equation (or theorem) implies that if the fluid is flowing (e.g., through a horizontal pipe) so that no change in gravitational potential energy occurs, then a decrease in fluid pressure is associated with an increase in fluid velocity.

Applications :

(i) Pumps.
(ii) Ejectors.
(iii) Siphon
(iv) Engine carburetors.
(v) Flow meters, e.g., venturi meter.
(vi) Automisers.
(vii) Bunsen burners.
(viii) Air foils.
(ix) Draft.
(x) Wind mills.
(xi) Aeroplane wings.

The working of all these is based on Bernoulli's principle/theorem.