

PHYSICS
USO1CPHY21
UNIT-3

Network Analysis and Bridges.
Part -2 Bridges and Their Applications

DC BRIDGES

INTRODUCTION

Bridges are used to measure the values of the electronic components. For example a Wheatstone bridge is used to measure the unknown resistance of a resistor. However they are also used to measure the unknown inductance, capacitance, admittance, conductance or any of the impedance parameters.

Besides this bridge circuits are also used in the precision measurements in some circuits and for the interfacing of transducers. Actually nowadays fully automatic bridges which electronically null a bridge to make precision measurements are also used.

WHEATSTONE BRIDGE: BASIC OPERATION

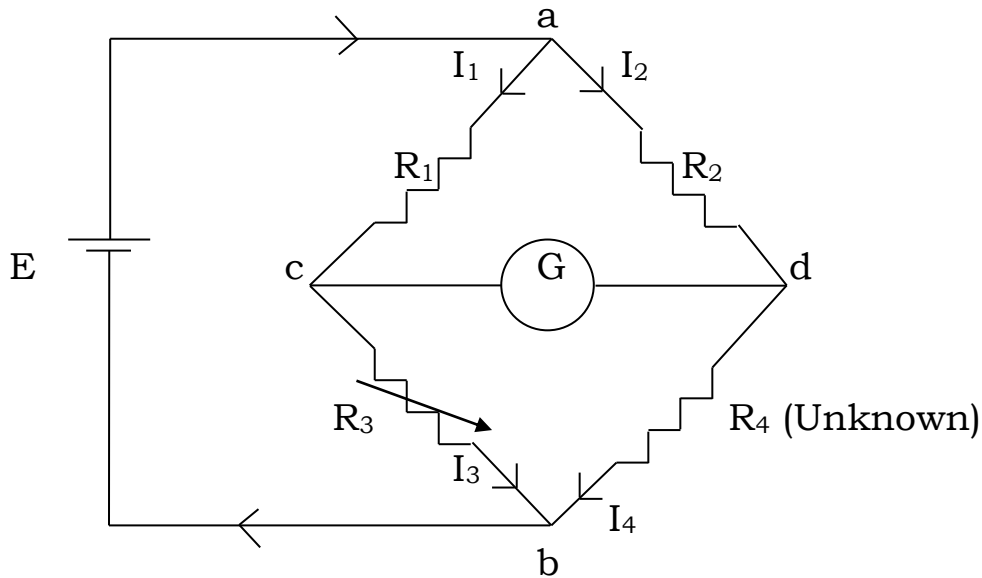


Fig. (1)

- The circuit diagram of Wheatstone bridge is shown in Fig. (1). the four arms of the bridge ac , ad , cb and db contains the four resistors R_1 ,

R_2 , R_3 and R_4 respectively. G is a galvanometer or the null detector. E is the source of EMF.

- I_1 , I_2 , I_3 and I_4 are the currents through the resistors R_1 , R_2 , R_3 and R_4 , respectively.
- When the current through galvanometer is zero, at that time terminals c and d are said to be at same potential with respect to point a i.e.,

$$E_{ac} = E_{ad} \tag{1}$$

- Hence the currents $I_1 = I_3$ and $I_2 = I_4$. This is called the balance of the bridge. And for this condition, we can write,

$$I_1 R_1 = I_2 R_2 \tag{2}$$

Where

$$I_1 = I_3 = \frac{E}{R_1 + R_3} \tag{3}$$

And

$$I_2 = I_4 = \frac{E}{R_2 + R_4} \tag{4}$$

- Substituting the values of I_1 and I_2 from equations (3) and (4) into (2), we get

$$\frac{E}{R_1 + R_3} R_1 = \frac{E}{R_2 + R_4} R_2$$
$$\frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4}$$

Therefore, $R_1(R_2 + R_4) = R_2(R_1 + R_3)$

$$R_1 R_4 = R_2 R_3 \tag{5}$$

- Equation (5) is called the balance equation (condition) of the bridge. Here, if R_4 is an unknown resistor, then its resistance R_x can be measured using the equation

$$R_1 R_x = R_2 R_3$$

$$R_x = \frac{R_2 R_3}{R_1} \quad (6)$$

- Here resistor R_3 is called the standard arm, whereas R_2 and R_1 are called the ratio arms.

MEASUREMENT ERRORS (LIMITATIONS OF THE WHEATSTONE BRIDGE CIRCUIT OR SOURCES OF ERRORS IN WHEATSTONE BRIDGE MEASUREMENTS)

- The major sources of errors in the Wheatstone bridge measurements are as follows:

- (1) Insufficient sensitivity of the null detector (galvanometer).
- (2) Heating effects of currents through the arm resistors: The currents flowing through the resistors produce heating effects in them and changes the resistance values. This affects the bridge balance. It leads to the I^2R losses in resistors. If an excess amount of current flows, the resistance values change permanently and this affects the subsequent measurements. This error becomes substantial particularly when low resistance values are to be measured. To avoid this, the current must be limited to a safe value.
- (3) Thermal EMF in the bridge circuit or the galvanometer circuit can also cause problems when low-value resistors are being measured. This limitation can be overcome using high sensitivity galvanometer having copper coil and copper suspension system in it.
- (4) Leads and Contacts: the resistance drops in the leads (wires) and contacts which are used in the external

bridge circuit affects the balance of bridge. This error can be minimized using a Kelvin bridge circuit.

AC BRIDGES AND THEIR APPLICATIONS

CONDITIONS FOR BRIDGE BALANCE

- An AC bridge consists of four bridge arms, a source of excitation and a null detector. The power source is an ac source and the null detector is usually a headphone.
- An AC bridge having impedances Z_1 , Z_2 , Z_3 and Z_4 in its arms is shown in the Fig. (1) below.

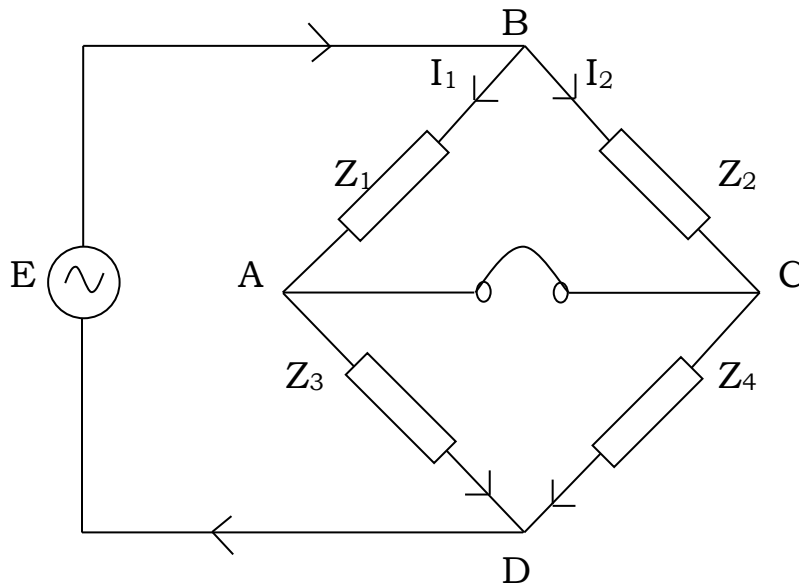


Fig. (1)

- When balance of the bridge is obtained the terminals A and C are said to be at same potential with respect to point B i.e.

$$E_{AB} = E_{BC} \tag{1}$$

- For this condition, we can write,

$$I_1 Z_1 = I_2 Z_2 \tag{2}$$

Where

$$I_1 = \frac{E}{Z_1 + Z_3} \tag{3}$$

and

$$I_2 = \frac{E}{Z_2+Z_4} \quad (4)$$

• Substituting the values of I_1 and I_2 from equations (3) and (4) into (2), we get

$$\frac{E}{Z_1+Z_3} Z_1 = \frac{E}{Z_2+Z_4} Z_2$$

$$\frac{Z_1}{Z_1+Z_3} = \frac{Z_2}{Z_2+Z_4}$$

Therefore, $Z_1(Z_2+Z_4) = Z_2(Z_1+Z_3)$

$$Z_1Z_4 = Z_2Z_3 \quad (5)$$

• Equation (5) is called the general equation for balance of AC Bridge. Similarly, we can also write this equation in terms of admittances(Y) instead of impedances as

$$Y_1Y_4 = Y_2Y_3 \quad (6)$$

• If the impedance is written in the form of $Z = \underline{Z} \theta$, then equation (5) can be written as

$$(Z_1/\theta_1) (Z_4/\theta_4) = (Z_2/\theta_2) (Z_3/\theta_3)$$

$$Z_1 Z_4 / (\theta_1 + \theta_4) = Z_2 Z_3 / (\theta_2 + \theta_3) \quad (7)$$

• Equation (7) shows two balance conditions. The first condition is that the magnitudes of the impedances satisfy the relationship shown by equation (5).

• It states that “The products of the magnitudes of impedances of the opposite arms must be equal”.

• The second condition requires that the phase angles of the impedances satisfy the relationship

$$\underline{\theta}_1 + \underline{\theta}_4 = \underline{\theta}_2 + \underline{\theta}_3 \quad (8)$$

- It states that “The sum of the phase angles of the opposite arms must be equal”.

MAXWELL BRIDGE

- The Maxwell bridge is used to measure an unknown inductance in terms of a known capacitance. The circuit for the Maxwell Bridge is shown in Fig. (1) below.

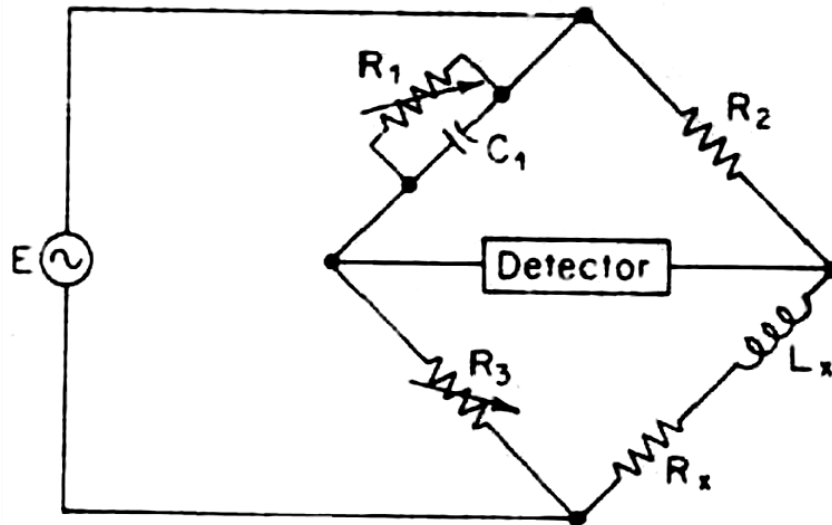


Fig. (1)

- As shown in Fig. (1), the ratio arm has parallel combination of resistance R_1 and a capacitance C_1 .
- The unknown arm contains unknown inductance L_x and resistance R_x .
- The balance condition of the bridge can be given by

$$Z_1 Z_x = Z_2 Z_3$$

$$Z_x = Z_2 Z_3 Y_1 \tag{1}$$

Where $Y_1 = 1/Z_1 = \text{admittance}$

- The bridge is balanced by first adjusting R_3 for inductive balance and then by adjusting R_1 for resistive balance.
- Adjustment of R_1 disturbs the inductive balance and R_3 needs to be modified and accordingly R_1 also is modified.

- The process gives a slow convergence and the balance slowly shifts towards the actual null point. This type of balance is called the ‘sliding balance’. The actual null point is obtained after few adjustments.
- From Fig. (1) we can write the impedance of the arms as

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1} + \frac{j}{X_1} = \frac{1}{R_1} + j\omega C_1 \quad (2)$$

$$Z_2 = R_2; \quad Z_3 = R_3 \quad (3)$$

$$Z_x = R_x + jX_L = R_x + j\omega L_x \quad (4)$$

$$R_x + j\omega L_x = R_2 R_3 \left(\frac{1}{R_1} + j\omega C_1 \right)$$

$$R_x + j\omega L_x = \left(\frac{R_2 R_3}{R_1} + j\omega C_1 R_1 R_2 \right)$$

- Equating the real parts of the above equation we get

$$R_x = \frac{R_2 R_3}{R_1}$$

- Equating the imaginary parts we get

$$L_x = R_2 R_3 C_1$$

• **Limitation of the Maxwell bridge** is that it is not suitable for the determination of the self-inductance of coils having quality factor $Q > 10$ or $Q < 1$. It is suitable for the coils having Q values in the range $1 < Q < 10$.

• This is because for high Q coils the phase angle is very nearly 90° (positive). This requires a capacitor having phase angle very nearly equal to 90° (positive).

• This situation demands that R_1 should be very large, which is impractical.

SCHERING BRIDGE

- A Schering Bridge is generally used to determine the unknown capacitance of a capacitor. However, it is sometimes also used to measure the insulating or dielectric properties of materials.
- The circuit arrangements for the bridge are given below in Fig. (1).

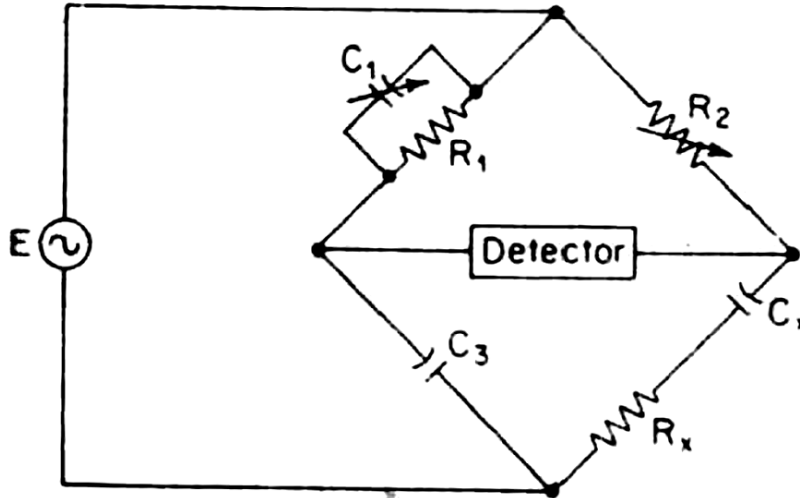


Fig. (1)

- Here, the ratio arm has the capacitor C_1 in parallel with resistor R_1 .
- The balance condition for the bridge is given by the equation

$$Z_x = Z_2 Z_3 Y_1 \quad (1)$$

- The impedances of the arms are given by

$$\frac{1}{Z_1} = Y_1 = \frac{1}{R_1} + \frac{j}{X_{C1}} = \frac{1}{R_1} + j\omega C_1 \quad ; \quad Z_2 = R_2; \quad (2)$$

$$Z_3 = -jX_{C3} = \frac{-j}{\omega C_3} \quad ; \quad Z_x = R_x - jX_{Cx} = R_x - \frac{j}{\omega C_x} \quad (3)$$

- Substituting the values from equations (2) and (3) into equation (1), we get

$$R_x - \frac{j}{\omega C_x} = R_2 \left(\frac{-j}{\omega C_x} \right) \left(\frac{1}{R_1} + j\omega C_1 \right)$$

$$R_x - \frac{j}{\omega C_x} = \frac{R_2 C_1}{C_3} - \frac{j R_2}{\omega C_3 R_1}$$

- Then equating the real and imaginary parts in the above equation, we get

$$R_x = R_2 C_1 / C_3 \quad \text{and} \quad C_x = C_3 R_1 / R_2 \quad (4)$$

- The power factor for the series RC combination is defined as the cosine of the phase angle of the circuit. It is given by

$$PF = R_x / X_x = \omega C_x R_x \quad (5)$$

- Similarly, the dissipation factor of a series RC circuit is defined as the cotangent of the phase angle of the circuit. It is given by

$$D = \omega C_1 R_1 \quad (6)$$

WIEN BRIDGE

- A Wien bridge is generally used to measure the frequency of the source. It is also extensively used in other applications like, harmonic distortion analyzer, in audio and HF oscillators as the frequency determining element, etc.
- As shown in Fig. (1) below, the Wien bridge consists of a series RC combination in one arm and a parallel RC combination in the adjoining arm.

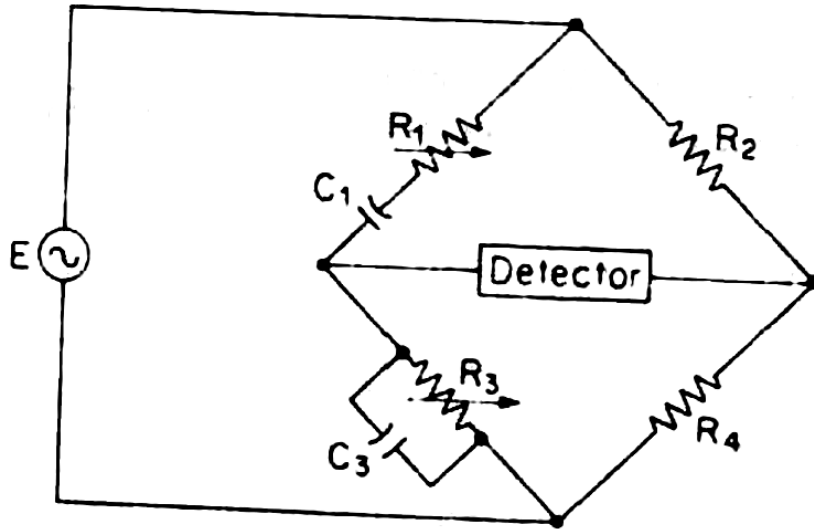


Fig. (1)

- The balance condition for the bridge is given by

$$Z_2 Z_3 = Z_1 Z_4$$

$$Z_2 = Z_1 Z_4 (1/Z_3) = Z_1 Z_4 Y_3 \quad (1)$$

- Now the impedances of the arms are given by the following equations

$$Z_1 = R_1 - jX_{C1} = R_1 - \frac{j}{\omega C_1} \quad ; \quad Z_2 = R_2 \quad (2)$$

$$1/Z_3 = Y_3 = \frac{1}{R_3} + \frac{j}{X_{C3}} = \frac{1}{R_3} + j\omega C_3 \quad ; \quad Z_4 = R_4 \quad (3)$$

- Substituting the values from equations (2) and (3) into equation (1), we get

$$R_2 = \left(R_1 - \frac{j}{\omega C_1} \right) R_4 \left(\frac{1}{R_3} + j\omega C_3 \right)$$

$$R_2 = \frac{R_1 R_4}{R_3} + j\omega C_3 R_1 R_4 - \frac{jR_4}{\omega C_1 R_3} + \frac{R_4 C_3}{C_1} \quad (4)$$

- Equating the real parts we get,

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1}$$

- Dividing by R_4 on both the sides we get

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \quad (5)$$

- Now equating the imaginary parts we get,

$$j\omega C_3 R_1 R_4 = \frac{jR_4}{\omega C_1 R_3} \quad (6)$$

$$\omega^2 = \frac{1}{C_1 C_3 R_1 R_3} \quad \text{where } \omega = 2\pi f$$

$$f = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}} \quad (7)$$

- If $R_1 = R_3 = R$ and $C_1 = C_3 = C$, then we get $R_2 = 2R_4$ and

$$f = \frac{1}{2\pi RC} \quad (8)$$

- Using the above equation, we can determine the frequency of the source. Also the bridge can be calibrated directly in terms of frequency.

MULTIPLE CHOICE QUESTIONS

- How many arms does a Wheatstone bridge possess?
(a) one (b) two **(c) four** (d) three
- When a Wheatstone bridge is balanced ideally the current through the galvanometer should be
(a) 0 A (b) 1 A (c) 1.5 A (d) 2 A
- Wheatstone bridge is used to find unknown
(a) resistance (b) reactance (c) inductance (d) capacitance
- In an ac bridge the null detector is usually
(a) a galvanometer (b) an ammeter
(c) a head phone (d) a voltmeter
- The inductive reactance of a coil(X_L) is given by the following equation
(a) $X_L = (1/2\pi)fL$ (b) $X_L = 2\pi/(fL)$ **(c) $X_L = 2\pi fL$** (d) $X_L = 2\pi f/L$
- The capacitive reactance of a capacitor (X_C) is given by the following equation
(a) $X_C = 1/fC$ **(b) $X_C = 1/(2\pi fC)$** (c) $X_C = 2\pi/fC$ (d) $X_C = 2\pi f/C$
- Maxwell bridge is generally used to measure the unknown
(a) resistance (b) reactance **(c) inductance** (d) capacitance
- Maxwell bridge is suitable for the coils having Quality factor in the range
(a) $0.1 < Q < 1$ **(b) $1 < Q < 10$** (c) $0.01 < Q < 0.1$ (d) $10 < Q < 1$
- Hay bridge is suitable for the coils having Quality factor
(a) $Q > 10$ (b) $Q < 10$ (c) $Q = 10$ (d) $Q < 1$

10. Scherring bridge is used to determine the unknown
(a) resistance (b) reactance (c) inductance **(d) capacitance**
11. Wien bridge is used to measure the unknown
(a) resistance (b) reactance **(c) frequency** (d) capacitance
12. When the balance condition is satisfied in an ac bridge the ratio of _____ of the adjoining arms are equal.
(a) resistances **(b) impedances** (c) reactances (d) phase angles
13. When the balance condition is satisfied in an ac bridge the sum of the _____ of the opposite arms are equal.
(a) resistances (b) impedances (c) reactances **(d) phase angles**
14. When the balance condition is satisfied in an dc bridge the ratio of the _____ of the adjoining arms are equal.
(a) resistances (b) impedances (c) reactances (d) phase angles
15. An inductor offers a ____ resistance to the high frequency current flowing through it.
(a) low (b) zero **(c) high** (d) infinite
16. A capacitor offers a ____ resistance to the high frequency current flowing through it.
(a) low (b) zero (c) high (d) infinite

SHORT QUESTIONS: (TWO MARKS)

- 1.** Draw the circuit diagram for the Wheatstone bridge and write its balance condition.
- 2.** Write the balance conditions for an AC bridge.
- 3.** Enlist the possible sources of errors in the Wheatstone bridge measurement.
- 4.** Explain the effect of heating of resistors and the thermo-emf on the Wheatstone bridge measurements.

5. Explain the effect of leads and contacts and the thermo-emf on the Wheatstone bridge measurements.
6. Write a note on limitations of Wheatstone bridge.
7. Discuss the limitations of Maxwell bridge.

LONG QUESTIONS

1. Discuss the basic operation of a Wheatstone bridge and derive its balance condition. **(4)**
2. Discuss in detail the various sources of errors associated with the Wheatstone bridge measurements. **(4)**
3. Derive the balance condition for an AC bridge with the help of the necessary circuit diagram. **(4)**
4. Describe the construction and working of Maxwell bridge and discuss the procedure to determine the unknown inductance of a coil using it. Also explain its limitations. **(7)**
5. Explain the construction and working of a Schering bridge in detail. **(5)**
6. Explain the construction and working of a Wien bridge in detail. **(5)**