V.P. & R.P.T.P. Science College, V.V.Nagar

Practice Test - 2020

B.Sc. Semester - IV Examination

Tuesday, $5^{th} May, 2020$

Time: 10 .00 to 1.00 pm | **Course Code: USC04CSTA22** | **M.Marks: 70**

(Probability Distributions)

Note: (i) Simple/Scientific calculator is allowed. (ii) Statistical table is allowed or provided on request.
(iii) Figures to the right indicate marks. (iv) Q.3 to 6 each sub question is of 5 marks

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Q.1	**Multiple Choice Questions**	(10×1)
(1)	Poisson distribution is a limiting case of (a) Binomial distribution (b) Negative Binomial distribution (c) Hypergeometric distribution (d) Both (a) and (b)	
(2)	If $P(x) = \frac{1}{11}$, $x = 0, 1, 2, \dots 10$ $and\ zero\ otherwise$, is the p.m.f of X then $V(X) =$ ________ (a) 6 (b) 11 (c) 10 (d) None of the above	
(3)	For a normal distribution (a) Mean = Median = Mode (b) Coefficient of skewness is zero (c) $P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9972$ (d) All of the above	
(4)	Let X have Bernoulli distribution with mean 0.4. What is the variance of $(2X - 3)$? (a) 0.24 (b) 0.48 (c) 0.6 (d) 0.96	
(5)	Let X be a chi square variate with 15 d.f, determine the value of k such that $P(X > k) = 0.025$. (a) 6.262 (b) 27.488 (c) 30.578 (d) 24.996	
(6)	The mean, median and mode for Binomial distribution will be equal when (a) $p = q$ (b) $p > q$ (c) $p < q$ (d) None	
(7)	Let X be a r.v. with pdf $f(x) = \frac{\alpha^\beta}{\lceil\beta} e^{-\alpha x} x^{\beta - 1}, x > 0, \alpha, \beta > 0$ $and\ zero\ otherwise$ If $E(X) = 20$ and $V(X) = 10$ then (α, β) is (a) $(2, 20)$ (b) $(2, 40)$ (c) $(4, 20)$ (d) $(4, 40)$	
(8)	We believe that 90% of the SYBSc Mathematics students of Sardar Patel University consider statistics to be an exciting subject. Suppose we randomly and independently selected 33 students from the SYBSc Mathematics students. Assume that the belief is true. Find the probability of observing 32 or more students who consider statistics to be an exciting subject. (a) 0.1442 (b) 0.1133 (c) 0.0309 (d) 0.8558	
(9)	If X_1 and X_2 are two i.i.d $N(0, 1)$ variates, then $P\left({X_1}^2 + {X_2}^2 \leq 2\right) =$ ________________ (a) e^{-1} (b) e^{-2} (c) $1 - e^{-1}$ (d) $1 - e^{-2}$	
(10)	Let X and Y be two independent normal variates with means -4 and 3 respectively and variances 4 and 9 respectively. What are the variance of $W = 3X - 2Y$? (a) 72 (b) 36 (c) 6 (d) 8.49	
Q.2	**Short Type Questions (Attempt Any Ten)**	(10×2)
(1)	If $X \sim N(5, 4)$, what is the prob. that $P(8 < Y < 13)$ where $Y = 2X + 1$? State clearly the result you had used to calculate the required probability.	
(2)	If $M_X(t) = \left(1 - \frac{t}{2}\right)^{-1}$, find the third cumulant.	
(3)	Let $X \sim P(m)$ and $P(X = 0) = 0.323$, find the value of m and use this to calculate $P(X = 3)$.	
(4)	A class contains 8 boys and 7 girls. The teacher selects 3 of the children at random and without replacement. Calculate the probability that the number of boys selected exceeds the number of girls selected.	
(5)	A r.v. X is uniformly distributed over (a, b). If $E(X) = -1/2$ and $V(X) = 3/4$, find the values of a and b.	
(6)	A new tax law is expected to benefit "middle income" families, those with incomes between \$20,000 to \$30,000. If family income follows normal distribution with mean \$25,000 and standard deviation \$10,000, what percentage of the	

population will benefit from the new tax law?

(7) Obtain the recurrence relation for the probabilities of Binomial distribution.

(8) It is known that the resistance of carbon resistors is normally distributed with mean 1200 and s.d 120 ohms. If 12 resistors are randomly selected from a shipment, what is the probability that the average resistance will be more than 1250 ohms?

(9) The following graph shows the uniform distribution of waiting times, in minutes, at Anand railway station. Find the area of the shaded region.

(10) You wish to draw a black Ace from deck of cards.
(i) What is the chance you draw your first black Ace on the 3rd draw?
(ii) What is the probability of drawing 2 or more black Aces in 3 independent draws of a card?

(11) A life insurance agent sells on average 3 life insurance policies per week. Use Poisson distribution to calculate the probability that in a given week he will sell (i) some policies (ii) 2 or more policies but less than 5 policies.

(12) Let X and Y be two independent random variables with moment generating functions $M_X(t) = e^{2t+4t^2}$ and $M_Y(t) = e^{t+6t^2}$. Determine the moment generating of $X + Y$ and identify and name the distribution of $X + Y$.

Q.3 (a) The probability mass function of a r.v. X is
$P(X = x) = \left(\frac{2}{3}\right)^x \left(\frac{1}{2}\right), x = 1, 2, 3, \ldots$ and zero otherwise, Find $V(X)$ and $P(X > 2)$

(b) A bowl contains 10 balls, of which 4 are red and 6 are white. Balls are randomly selected with replacement from the bowl until 4 red balls have been selected. Let X be the number of white balls drawn before the fourth red ball is selected. Find the mean and variance of X. Determine $P(X = 6)$.

OR

Q.3 (a) Define Hypergeometric distribution. Obtain Binomial distribution as a limiting case of Hypergeometric distribution.

(b) It was claimed that 1 out of 100 dentists recommend Colgate sensitive to his patients for sensitivity of teeth. Suppose that the claim is true. If 120 dentists are selected independently and at random, let X be the number of dentists who recommend Colgate sensitive to his/her patients. (i) How is X distributed? (ii) Give the mean and variance of X. (iii) Determine $P(X \geq 4)$

Q.4 (a)

(b) The distribution of 1000 examinees according to marks percentage is given below:

% of marks	Less than 40	$40 - 75$	75 or above
No. of examinees	430	420	150

Assuming the marks percentage to follow a normal distribution, calculate the mean and standard deviation of marks. If 40% examinees are to fail, what should be the passing marks?

OR

Q.4 (a) Let $X \sim U(-2, 2)$. Show that all the odd order moments are zero. Obtain an expression for even order moments.

(b) A r.v. X has pdf $f(x) = kx^4(1 - x)^4, 0 < x < 1$ *and zero otherwise*. Determine the value of k.
Find $P(|X - \mu| \leq 2\sigma)$ where μ and σ are the mean and s.d of r.v. X.

Q.5 (a) About 10% of the population is left – handed. Use the normal approximation to approximate the probability that in a class of 150 students, (i) at least 25 of them are left – handed. (ii) between 15 and 20 are left – handed.

(b) Suppose that the weights in lbs of American adult can be represented by a normal variate with mean 150 lbs and variance 900 lb^2. An elevator containing a sign "Maximum 12 people can safely carry 2000 lbs". Find the probability that 12 people will not overload elevator. State and prove the result you have used to calculate the required probability.

OR

Q.5 (a) Prove that the sum of two independent binomial variates is also a binomial variate. Is the difference of two binomial variates is binomial?

(b) Marks obtained by certain students are assumed to be normally distributed with mean 65 and variance 25. If three students are taken at random, what is the probability that exactly two of them will have marks over 70?

Q.6 (a)	**Do as directed:** **(i) If $X_1, X_2, \dots, X_{10}$ be a random sample from a standard normal distribution. Find the numbers a and b such that** $P\left(a \le \sum_{i=1}^{10} X_i^2 \le b\right) = 0.95$ **(ii) Let $X_1, X_2, \dots, X_{10}$ be a random sample of size $n = 10$ from a normal distribution with variance $\sigma^2 = 0.8$. Find two positive numbers a and b such that $P(a \le S^2 \le b) = 0.90$ where** $S^2 = \frac{1}{9}\sum_{i=1}^{10}(X_i - \bar{X})^2$
(b)	**If S_1^2 and S_2^2 are the variances of independent random samples of size $n_1 = 10$ and $n_2 = 15$ from normal populations with equal variances, find a constant $'k'$ so that $P\left(\frac{S_1^2}{S_2^2} > k\right) = 0.95$ where** $S_1{}^2 = \frac{1}{9}\sum_{i=1}^{10}(X_i - \bar{X})^2$ *and* $S_2{}^2 = \frac{1}{14}\sum_{i=1}^{15}(Y_i - \bar{Y})^2$
	OR
Q.6 (a)	**Let $\overline{X_1}$ and $\overline{X_2}$ be the means of samples of sizes $n_1 = 4$ and $n_2 = 9$ from two normal populations with means $\mu_1 = 2, \mu_2 = 4$ and variances $\sigma_1^2 = 6$ and $\sigma_2^2 = k$. If $P(\overline{X_1} - \overline{X_2} > 8) = 0.0228$, then what is the value of $'k'$?**
(b)	**Let $\bar{X}$ and S^2 be the sample mean and variance associated with a r.s. of size $n = 12$ from a normal distribution with mean μ and variance 144.** **(i) Find the constants a and b so that $P(a \le S^2 \le b) = 0.99$** **$(ii)$Find a constant k so that $P\left(-k \le \frac{\bar{X}-\mu}{S} \le k\right) = 0.99$, where $\bar{X} = \frac{1}{n}\sum Xi$ *and* $S^2 = \frac{1}{n-1}\sum(X_i - \bar{X})^2$**