CONDUCTION

Heat transfer deals with the study of rates at which exchange of heat takes place between a hot source and a cold receiver. In process industries there are many operations which involve transfer of energy in the form of heat, e.g., evaporation, distillation, drying, etc. and also chemical reactions carried on a commercial scale take place with evolution or absorption of heat. It is also necessary to prevent the loss of heat from a hot vessel or a pipe system to the ambient air. In all these cases, the major problem is that of transfer of heat at the desired rate. The knowledge of laws of heat transfer, mechanisms of heat transfer and process heat transfer equipments is of great importance from a stand point of controlling the flow of heat in the desired manner.

It is well established fact that if two bodies at different temperatures are brought into thermal contact, heat flows from a hot body to a relatively cold body (**second law of thermodynamics**). The net flow of heat is always in the direction of decrease in the temperature. Thus, heat is defined as *a form of energy which is in transit between a hot source and a cold receiver*. The transfer of heat solely depends upon the temperatures of the two bodies/substances/parts of a system. In other words, temperature can be termed as the level of thermal (heat) energy, i.e., high temperature of a body is the indication of high level of heat energy content of the body.

Heat may flow by any one or more of the three basic mechanisms, namely, conduction, convection, and radiation. We will first see these three modes of heat transfer in brief and then we will consider heat conduction through solids in detail.

Conduction : It is the transfer of heat from one part of a body to the another part of the same body or from one body to another which is in physical contact with it, without appreciable displacement of particles of a body. Conduction is restricted to the flow of heat in solids. Examples of conduction : Heat flow through the brick wall of a furnace, the metal sheet of a boiler and the metal wall of a heat exchanger tube.

Convection : It is the transfer of heat from one point to another point within a fluid (gas or liquid) by mixing of hot and cold portions of the fluid. It is attributed to the macroscopic motion of fluid. Convection is restricted to the **flow of heat in fluids** and is closely associated with fluid mechanics. In **natural convection**, the fluid motion results from the difference in densities of the warmer and cooler fluid arising from the temperature difference in the fluid mass. In **forced convection**, the fluid motion is produced by mechanical means such as an agitator, a fan or pump. Examples of heat transfer mainly by convection are : heating of room by means of a steam radiator, heating of water in cooking pans, heat flow to a fluid pumped through a heated pipe.

Radiation : Radiation refers to the transfer of heat energy from one body to another through space, not in contact with it, by electromagnetic waves. Examples of heat transfer by radiation mode are : the transfer of heat from the sun to the earth and the loss of heat from an unlagged steam pipe to the ambient air.

Conduction as well as convection occurs only in the presence of material medium whereas radiation can occur even in vacuum and no material medium is required for heat flow by radiation. It is observed that heat flow by conduction is slow, faster by convection and the fastest by radiation mode.

CONDUCTION :

It is our common observation that when some material object is heated at one of its locations, then in a short while its remaining parts also get heated. This shows that heat flows through the material object from a high temperature region to a low temperature region. The flow of heat in this manner is called as heat conduction or simply conduction, wherein the particles of object participate in the process but they do not move bodily from the hot or high temperature region to the cold or low temperature region.

Conduction is the mode of heat transfer in which a material medium transporting the heat remains at rest. The heat conduction occurs by the migration of molecules and more effectively by the collision of the molecules vibrating around relatively fixed positions. In liquids and solids where little or no migration occurs, heat is transferred by the collision of vibrating molecules. [The molecules of a substance are always in a state of vibration. When the substance is heated at one of its locations, the molecules of that location receive energy and they begin to vibrate with larger amplitudes and as a result of increase in their amplitude, they will collide with the neighbouring molecules and in the process they transfer a part of their energy to the neighbouring molecules. This process occurs repeatedly and thus results in heat flow from one molecule to another along the heat flow path i.e. through the substance.]

Conduction refers to *the mode of heat transfer in which the heat flow through the material medium occurs without actual migration of particles of the medium from a region of higher temperature to a region of lower temperature.*

It is a fact that conduction occurs in solids, liquids and gases but pure conduction is found to present only in solids, with gases and liquids it is present with convection, so we will consider here heat conduction in solids for better understanding of conduction mechanism as convection is not present in solids.

In this chapter, we restrict our discussion to **steady state unidirectional heat conduction in solids.**

By *steady state heat flow* we mean that the situation of heat flow in which the temperature at any location along the heat flow path does not vary with time and the rate of heat transfer does not vary with time. In other words, it is the heat flow under conditions of constant temperature distribution-temperature is a function of location only, i.e., temperature varies with location but not with time. Hence, steady state heat conduction is the heat transfer by conduction under conditions of constant temperature distribution.

By unidirectional or one dimensional heat flow we mean that the flow of heat occurring only in one direction, i.e., along one of the axes of the respective coordinate system used. (For example, say along the x-direction in case of a Cartesian co-ordinate system).

Fourier's Law :

The physical law governing the transfer of heat through a uniform material (whenever a temperature difference exists) by a conduction mode was given by the French scientist : Joseph Fourier.

Fourier's law states that *the rate of heat flow by conduction through a uniform (fixed) material is directly proportional to the area normal to the direction of the heat flow and the temperature gradient in the direction of the heat flow.*

Mathematically, the Fourier's law of heat conduction for steady state heat flow is given by

$$Q \propto A[-dT/dn] \quad \ldots (2.1)$$

$$Q = -kA\,[dT/dn] \quad \ldots (2.2)$$

where Q is the rate of heat flow/transfer in watts (W), A is the area normal to the direction of heat flow in m^2, T is the temperature in K, n is the distance measured normal to the surface, i.e., the length of conduction path along the heat flow in m, dT/dn is the rate of change of temperature with distance measured in the direction of heat flow (called as temperature gradient) in K/m. k is a constant of proportionality and is called the thermal conductivity. It is the characteristic property of a material through which heat flows.

The negative sign is incorporated in equation (2.2) because the temperature gradient is negative (since with an increase in n there is a decrease in T, i.e., temperature decreases in the direction of heat flow) and it makes the heat flow positive in the direction of temperature decrease.

The Fourier's law for a steady state unidirectional (say in the x-direction) heat conduction then becomes

$$Q = -kA\,[dT/dx] \quad \ldots (2.3)$$

$$q = Q/A = -k\,[dT/dx] \quad \ldots (2.4)$$

where Q is the rate of heat flow, i.e., heat flow per unit time in W, and q is the heat flux, i.e., the rate of heat flow per unit area in W/m^2 (in the x-direction). In further discussion we will make use of Equation (2.3). The **Fourier's law** [equation (2.3)] is a fundamental differential equation of heat transfer by conduction. It is simply a definition of k.

[The heat flux is defined as the amount of heat transfer per unit area per unit time or the rate of heat transfer per unit area, Q/A.]

One Dimensional Steady State conduction :

Steady state heat conduction is a simpler case in the sense that the temperature does not vary with time. T is independent of time and is a function of position in the conducting solid.

One dimensional heat conduction implies that the temperature gradient exists only in one direction which makes the heat flow unidirectional. The cases of heat flow through a slab (plane wall), a circular cylinder, a sphere and long fins can be analysed by a one dimensional steady state conduction. In the discussion to follow we will treat heat flow to be in x-direction only.

In discussion to follow we assume that k does not vary with temperature.

Plane wall (slab) of uniform thickness :

The heat flow through the wall of a stirred tank containing a hot or cold fluid or the wall of a large furnace can be examples of one dimensional heat flow. Consider a plane/flat wall as shown in Fig. 2.1.

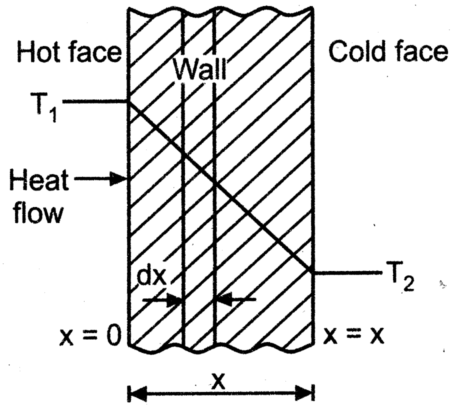

Fig. 2.1 : Conduction through a plane wall

Consider that the wall is made of a material of thermal conductivity k and is of uniform thickness (x) and constant cross-sectional area (A). Assume that k is independent of temperature and the area of wall is very large in comparison with the thickness so that the heat losses from the edges are negligible. A hot face is at temperature T_1 and a cold face is at temperature T_2 and both are isothermal surfaces. The direction of heat flow is perpendicular to the wall and T varies in the direction of x-axis.

As in steady state, there can be neither accumulation nor depletion of heat within the plane wall, Q is constant along the path of heat flow. The usual use of Fourier's law requires that the differential equation (2.3) be integrated over the entire path from x = 0 to x = x (total thickness of the wall) as we normally know temperatures only at the faces.

$$Q = -kA\left[\frac{dT}{dx}\right]$$

$$Q\,dx = -k \cdot A\,dT \qquad \ldots (2.5)$$

The variables in Equation (2.5) are x and T.

$$Q\int_0^x dx = -k \cdot A\int_{T_1}^{T_2} dT \qquad \ldots (2.6)$$

$$Q \cdot x = -kA\,(T_2 - T_1) \qquad \ldots (2.7)$$

Rearranging, we get

$$Q = \frac{kA\,(T_1 - T_2)}{x} \quad \ldots (2.8)$$

$$Q = \frac{k \cdot A}{x} \cdot \Delta T \quad \ldots (2.9)$$

where $\Delta T = (T_1 - T_2)$

$$Q = \frac{\Delta T}{x/kA} = \frac{\Delta T}{R} \quad \ldots (2.10)$$

where R (= x/kA) is the thermal resistance (of the wall material of thickness x), Q is the rate of heat flow (rate of heat transfer) and ΔT is the driving force for heat flow.

Equation (2.10) equates the rate of heat flow to the ratio of driving force to thermal resistance.

The reciprocal of resistance is called the conductance, which for heat conduction is :

$$\text{Conductance} = 1/R = 1/(x/kA) = k.A/x \quad \ldots (2.11)$$

Both the resistance and conductance depend upon the dimensions of a solid as well as on the thermal conductivity, a property of the material.

When k varies linearly with T (Equation 2.12), Equation (2.10) can be used rigorously by taking an average value $\bar{k}$ for k. $\bar{k}$ may be obtained either by using the arithmetic average of the individual values of k at surface temperatures T_1 and T_2 [$\bar{k} = (k_1 + k_2)/2$] or by calculating the arithmetic average of temperatures [$(T_1 + T_2)/2$] and using the value of k at that temperature. One can take linear variation of k with T under integration sign and integrate the equation.

Thermal Conductivity :

The proportionality constant 'k' given in Equation (2.2) is called as the thermal conductivity. It is a characteristic property of the material through which heat is flowing and varies with temperature. It is one of the so called transport properties of the material (like viscosity, μ).

Thermal conductivity is *a measure of the ability of a substance to conduct heat.* Larger the value of k, higher will be the amount of heat conducted by that substance.

Thermal conductivity is *the quantity of heat passing through a quantity of material of unit thickness with unit heat flow area in unit time when a unit temperature difference is maintained across the opposite faces of material.*

If Q is measured in watts (W ≡ J/s), A in m², x in m and T in K, then the unit of k (thermal conductivity) in the SI system is W/(m.K).

$$Q = -kA\,(dT/dx)$$

$$k = \frac{-Q \cdot dx}{A \cdot dT},\ \frac{W \cdot m}{(m^2 \cdot K)}$$

$$= \frac{-Q \cdot dx}{A \cdot dT},\ W/(m \cdot K) \equiv J/(s.m.K)$$

Thermal conductivity depends upon the nature of material and its temperature. Thermal conductivities of solids are higher than that of liquids and liquids are having higher thermal conductivities than for gases.

In general, thermal conductivity of gases ranges from 0.006 to 0.6 W/(m·K) while that of liquids ranges from 0.09 to 0.7 W/(m·K). Thermal conductivity of metals varies from 2.3 to 420 W/(m·K). The materials having higher values of thermal conductivity are referred to as **good conductors** of heat, e.g., metals. The best conductor of heat is silver [k = 420 W/(m·K)] followed by red copper [k = 395 W/(m·K)], gold [k = 302 W/(m·K)] and aluminium [k = 210 W/(m·K)]. The materials having low values of thermal conductivity [less than 0.20 W/(m·K)] are called as and used as **heat insulators** to minimise the rate of heat flow. e.g. asbestos, glass wool, cork, etc.

For small temperature ranges, thermal conductivity may be taken as constant but for large temperature ranges, it varies linearly with temperature and the variation of the thermal conductivity with temperature is given by the relationship

$$k = a + bT \quad \text{... (2.12)}$$

where a and b are empirical constants and T is the temperature in K.

Compound resistances in series / Heat conduction through a composite plane wall :

When a wall is formed out of a series of layers of different materials, it is called as a **composite wall.**

Consider a flat wall constructed of a series of layers of three different materials as shown in Fig. 2.2. Let k_1, k_2 and k_3 be the thermal conductivities of the materials of which layers are made. Let thicknesses of the layers be x_1, x_2 and x_3 respectively.

Let ΔT_1 be the temperature drop across the first layer, ΔT_2 that across the second layer and ΔT_3 that across/over the third layer. Let ΔT be the temperature drop across the entire composite wall.

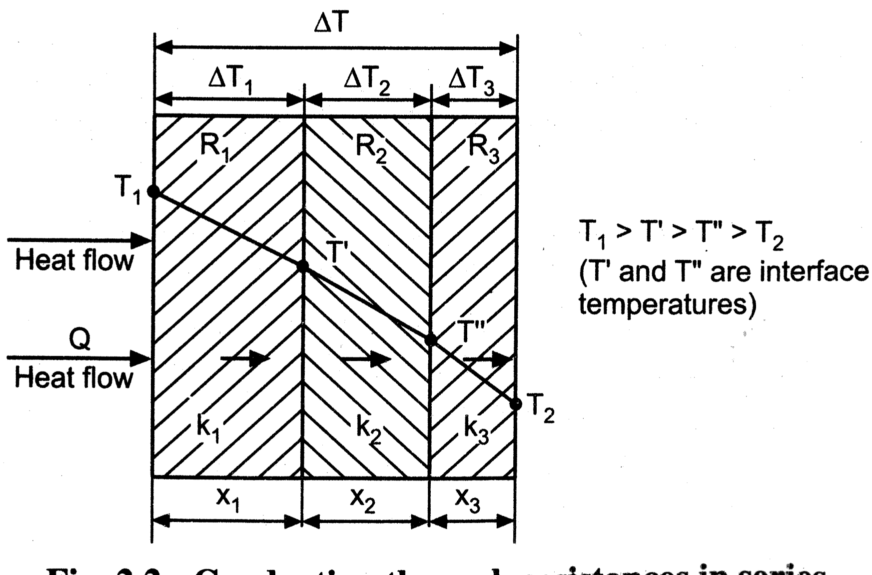

Fig. 2.2 : Conduction through resistances in series

Let T_1, T', T" and T_2 be the temperatures at the faces of the wall as shown in Fig. 2.2. T_1 is the temperature of the hot face and T_2 is the temperature of the cold face.

Assume further that the layers are in excellent thermal contact.

Furthermore, let the area of the composite wall, at right angles to the plane of illustration, be A.

Overall temperature drop is related to individual temperature drops over the layers by equation :

$$\Delta T = \Delta T_1 + \Delta T_2 + \Delta T_3 \quad \text{... (2.13)}$$

It is desired to derive an equation / formula giving the rate of heat flow through a series of resistances.

Rate of heat flow through the layer-1, i.e., through the material of thermal conductivity k_1 is given by

$$Q_1 = \frac{k_1 A}{x_1}(T_1 - T') \quad \text{... (2.14)}$$

$$(T_1 - T') = \frac{Q_1}{(k_1 A/x_1)} \quad \text{... (2.15)}$$

$$\Delta T_1 = T_1 - T' \quad \text{... (2.16)}$$

$$\therefore \quad \Delta T_1 = \frac{Q_1}{(k_1 A/x_1)} \quad \text{... (2.17)}$$

Similarly for layer-2

$$\Delta T_2 = (T' - T'') = \frac{Q_2}{(k_2 A/x_2)} \quad \text{... (2.18)}$$

and for layer-3 :

$$\Delta T_3 = (T'' - T_2) = \frac{Q_3}{(k_3 A/x_3)} \quad \text{... (2.19)}$$

Adding Equations (2.17), (2.18) and (2.19), we get

$$\Delta T_1 + \Delta T_2 + \Delta T_3 = \frac{Q_1}{(k_1 A/x_1)} + \frac{Q_2}{(k_2 A/x_2)} + \frac{Q_3}{(k_3 A/x_3)} = \Delta T \quad \text{... (2.20)}$$

Under steady state conditions of heat flow, all the heat passing through the layer-1 (first resistance) must pass through the layer-2 (second resistance) and in turn pass through the layer-3 (third resistance), therefore Q_1, Q_2 and Q_3 must be equal and can be denoted by Q. Thus, using this fact, Equation (2.20) becomes

$$\frac{Q}{(k_1 A/x_1)} + \frac{Q}{(k_2 A/x_2)} + \frac{Q}{(k_3 A/x_3)} = \Delta T \quad \text{... (2.21)}$$

$$Q\left[\frac{1}{(k_1 A/x_1)} + \frac{1}{(k_2 A/x_2)} + \frac{1}{(k_3 A/x_3)}\right] = \Delta T \quad \text{... (2.22)}$$

$$\therefore \quad Q = \frac{\Delta T}{\left[\frac{1}{k_1 A/x_1} + \frac{1}{k_2 A/x_2} + \frac{1}{k_3 A/x_3}\right]} \quad \text{... (2.23)}$$

$$Q = \frac{\Delta T}{\left[\frac{x_1}{k_1 A} + \frac{x_2}{k_2 A} + \frac{x_3}{k_3 A}\right]} \quad \text{... (2.24)}$$

Let R_1, R_2 and R_3 be the thermal resistances offered by the layer-1, 2 and 3 respectively. R_1, R_2 and R_3 are given as :

$$R_1 = x_1/k_1A \quad \text{... (2.25)}$$

$$R_2 = x_2/k_2A \quad \text{... (2.26)}$$

and

$$R_3 = x_3/k_3A \quad \text{... (2.27)}$$

With this Equation (2.24) becomes

$$Q = \frac{\Delta T}{R_1 + R_2 + R_3} \quad \text{... (2.28)}$$

If R is the overall resistance, then for resistances in series, we have :

$$R = R_1 + R_2 + R_3 \quad \text{... (2.29)}$$

Equation (2.28) becomes :

$$Q = \frac{\Delta T}{R} \quad \text{... (2.30)}$$

Equation (2.30) is used to calculate the rate of heat flow/heat transfer. It is the ratio of the overall temperature drop (driving force) to the overall resistance of the composite wall.

Equation (2.30) is the same as the equation for the rate of any process :

$$\text{Rate of transfer process} = \frac{\text{Driving force}}{\text{Resistance}}$$

One can calculate the temperatures at the interfaces of layers of which the wall is made by making use of the following relation :

$$\frac{\Delta T}{R} = \frac{\Delta T_1}{R_1} = \frac{\Delta T_2}{R_2} = \frac{\Delta T_3}{R_3} \quad \text{... (2.31)}$$

Based upon the thickness and thermal conductivity of a layer, temperature drop in that layer may be large or small fraction of the total temperature drop. A thin layer with a low thermal conductivity value may cause a much larger temperature drop and a steeper thermal gradient than a thick layer having a high thermal conductivity.

Heat flow through a cylinder :

Consider a thick walled hollow cylinder as shown in Fig. 2.3 of inside radius r_1, outside radius r_2 and length L. Let k be the thermal conductivity of the material of cylinder.

Let the temperature of the inside surface be T_1 and that of the outside surface be T_2. Assume that $T_1 > T_2$, therefore heat flows from the inside of the cylinder to the outside. It is desired to calculate the rate of heat flow for this case.

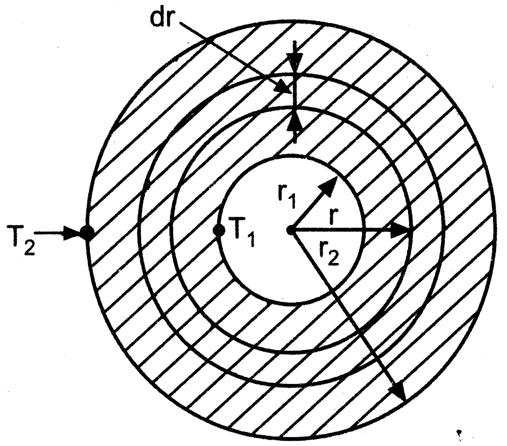

Fig. 2.3 : Heat flow through thick walled cylinder

Consider a very thin cylinder (cylindrical element), concentric with the main cylinder, of radius r, where r is in between r_1 and r_2. The thickness of wall of this cylindrical element is dr.

The rate of heat flow at any radius r is given by

$$Q = -k\, 2\pi rL \left(\frac{dT}{dr}\right) \qquad \dots (2.32)$$

Equation (2.32) is similar to Equation (2.3). Here the area perpendicular to the heat flow is $2\pi rL$ and dx of Equation (2.3) is equal to dr.

Rearranging Equation (2.32), we get

$$\frac{dr}{r} = \frac{-k\,(2\pi L)}{Q}\, dT \qquad \dots (2.33)$$

the only variables in Equation (2.33) are r and T (assuming k to be constant).

Integrating Equation (2.33) between the limits

when $r = r_1$, $T = T_1$

and when $r = r_2$, $T = T_2$ gives

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{-k\,(2\pi L)}{Q} \int_{T_1}^{T_2} dT \qquad \dots (2.34)$$

$$\ln r_2 - \ln r_1 = \frac{-k\,(2\pi L)\,(T_2 - T_1)}{Q} \qquad \dots (2.35)$$

$$\ln (r_2/r_1) = \frac{k\,(2\pi L)\,(T_1 - T_2)}{Q} \qquad \dots (2.36)$$

The rate of heat flow through a thick walled cylinder is

$$\therefore \quad Q = \frac{k\,(2\pi L)\,(T_1 - T_2)}{\ln (r_2/r_1)} \qquad \dots (2.37)$$

Equation (2.3) can be used to calculate the flow of heat through a thick walled cylinder.

It can be put into a more convenient form by expressing the rate of heat flow as :

$$Q = \frac{k\,(2\pi r_m L)\,(T_1 - T_2)}{(r_2 - r_1)} \qquad \dots (2.38)$$

where r_m is the logarithmic mean radius and is given by

$$r_m = \frac{(r_2 - r_1)}{\ln (r_2/r_1)} = \frac{(r_2 - r_1)}{2.303 \log (r_2/r_1)} \quad \dots (2.39)$$

$$A_m = 2\pi r_m L \quad \dots (2.40)$$

A_m is called the logarithmic mean area.

Equation (2.38) becomes :

$$Q = \frac{k A_m (T_1 - T_2)}{(r_2 - r_1)} \quad \dots (2.41)$$

$$Q = \frac{(T_1 - T_2)}{(r_2 - r_1) / k A_m} = \frac{\Delta T}{R}$$

where $R = (r_2 - r_1) / kA_m$

The RHS of Equation (2.39) is known as the logarithmic mean and in the particular case of Equation (2.39), r_m is known as the logarithmic mean radius. It is the radius which when applied to the integrated equation for a flat wall, will give the correct rate of heat flow through a thick-walled cylinder.

In case of thin-walled cylinders, the logarithmic mean is less convenient than the arithmetic mean, and the arithmetic mean is used without an appreciable error.

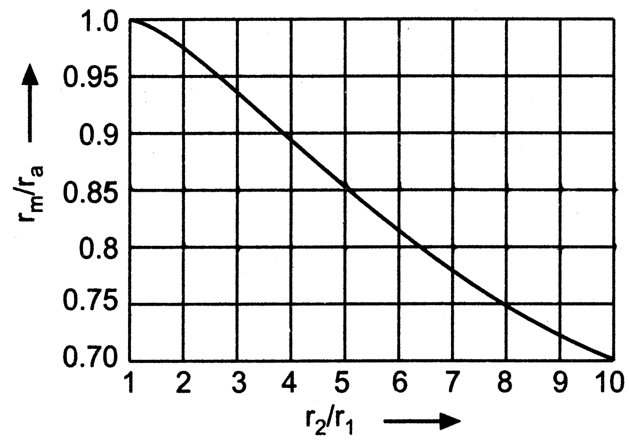

Fig. 2.4 : Relation between logarithmic and arithmetic means

Heat flow through a sphere :

Consider a hollow sphere of inner radius r_1 and outer radius r_2. Let T_1 be the temperature at the inner surface and T_2 be the temperature at the outer surface. Assume that $T_1 > T_2$, so that heat will flow from inside to outside.

Consider a spherical element at any radius r (between r_1 and r_2) of thickness dr.

Then rate of heat flow according to Fourier's law is given by

$$Q = -k (4\pi r^2) \frac{dT}{dr} \quad \dots (2.42)$$

where $A = 4\pi r^2$ = area of heat transfer

k = thermal conductivity of a material of which sphere is made

Rearranging Equation (2.42), we get

$$\frac{dr}{r^2} = \frac{-4\pi k}{Q} \, dT \qquad \ldots (2.43)$$

Integrating Equation (2.43) between the limits :

when $r = r_1$, $T = T_1$

and $r = r_2$, $T = T_2$

$$\int_{r_1}^{r_2} \frac{dr}{r^2} = \frac{-4\pi k}{Q} \int_{T_1}^{T_2} dT \qquad \ldots (2.44)$$

$$\left[-\frac{1}{r}\right]_{r_1}^{r_2} = \frac{-4\pi k}{Q} (T_1 - T_2) \qquad \ldots (2.45)$$

$$\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{4\pi k}{Q} (T_1 - T_2) \qquad \ldots (2.46)$$

Rearranging, we get

$$Q = \frac{4\pi k (T_1 - T_2)}{\left[\frac{1}{r_1} - \frac{1}{r_2}\right]} \qquad \ldots (2.47)$$

$$Q = \frac{4\pi \, r_1 r_2 \, k (T_1 - T_2)}{(r_2 - r_1)} \qquad \ldots (2.48)$$

$r_m = \sqrt{r_1 r_2}$ = mean radius which is geometric mean for sphere.

∴ Equation (2.48) becomes :

$$Q = \frac{4\pi \, r_m^2 \, k (T_1 - T_2)}{(r_2 - r_1)} \qquad \ldots (2.49)$$

Thermal Insulation :

Process equipments such as a reaction vessel, reboiler, distillation column, evaporator, etc. or a steam pipe will lose heat to the atmosphere by conduction, convection and radiation. In such cases, the conservation of heat that is usually of steam and coal is an economic necessity and therefore some form of lagging should be applied to the hot surfaces. In furnaces, the surface temperature is reduced substantially by making use of a series of insulating bricks that are poor conductors of heat.

Insulation is necessary (i) to prevent an excessive flow of heat to the surroundings from process units and pipelines in which heat is generated, stored or conveyed at temperatures above the surrounding temperature, (ii) to prevent an excessive flow of heat from the outside to materials which must be kept at temperatures below that of the surroundings, (iii) to provide for protection of personnel from skin damage through contact with very hot and very cold surfaces (to provide a safe work environment) and (iv) to provide comfortable/acceptable working environment. The working environment in the viscinity of process units and pipelines carrying hot or cold streams can become uncomfortable and unacceptable, if insulation is not provided. In a chemical plant, steam is transported to process equipments, as per requirement, through steam lines. If the steam lines are not insulated, then the loss of heat from these lines to the ambient air may result in the condensation of steam, thus lowering the quality of steam and creating operational problems in the equipments in which the steam is admitted.

The important requirements of an insulating material are as follows :

(i) It should have a low thermal conductivity.

(ii) It should withstand working temperature range.

(iii) It should have a sufficient durability and an adequate mechanical strength. This includes resistance to moisture and the chemical environment.

(iv) It should be easy to apply, non-toxic, readily available, inexpensive (low basic material cost, installation cost and maintenance cost).

(v) It should not create a fire hazard.

Cork [k = 0.025 W/(m·K)], asbestos (k = 0.10), glass wool (k = 0.024), 85 percent magnesia (k = 0.04) are commonly employed lagging materials in industry. Cork is common in refrigeration plants. 85% magnesia with asbestos, glass wool are widely used for lagging steam pipes. Thin aluminium sheeting is often used to protect the lagging.

The optimum thickness of insulation :

The optimum thickness of an insulation is obtained by a purely economic approach. The greater the thickness, the lower the heat loss and the greater the initial cost of insulation and the greater the annual fixed charges (maintenance and depreciation).

It is obtained by a purely economic approach. Increasing the thickness of an insulation reduces the loss of heat and thus gives saving in operating costs; but at the same time, cost of insulation will increase with thickness. The optimum thickness of an insulation is the one at which the total annual cost (the sum of the cost of heat lost and annual fixed charges) of the insulation is minimum.

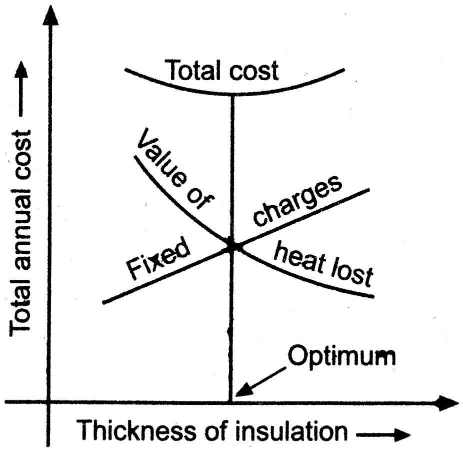

Fig. 2.5 : Optimum thickness of insulation

Note : Discussion on **systems with variable k** and **critical radius of insulation** is given at the end of this chapter.

SOLVED EXAMPLES

Example 2.1 : *Calculate the rate of heat loss Q, through a wall of red brick [k = 0.70 W/(m·K)] 5 m in length, 4 m in height and 250 mm in thickness, if the wall surfaces are maintained at 373 K (100° C) and 303 K (30° C) respectively.*

Solution : Mean area of heat transfer = $A = 5 \times 4 = 20\ m^2$

Thickness of brick wall = x = 250 mm = 0.25 m

Temperature difference = $\Delta T = 373 - 303 = 70$ K

Thermal conductivity of red brick = k = 0.70 W/(m·K)

The rate of heat loss is

$$Q = k \cdot A \left[\frac{\Delta T}{x}\right] = 0.70 \times 20 \times \left[\frac{70}{0.25}\right] = \mathbf{3920\ W} \quad \text{... Ans.}$$

Example 2.2 : *Estimate the heat loss per m² of the surface through a brick wall 0.5 m thick when the inner surface is at 400 K (127° C) and the outside surface is at 310 K (37° C). The thermal conductivity of the brick may be taken as 0.7 W/(m·K).*

Solution : Let the area of heat transfer be 1 m².

We have :
$$Q = \frac{k \cdot A\,(T_1 - T_2)}{x}$$

where

$k = 0.7$ W/(m·K)

$A = 1\ m^2$

$T_1 = 400$ K, $T_2 = 310$ K, $x = 0.5$ m

The rate of heat loss per 1 m² area is

$$Q = \frac{0.7 \times 1.0 \times (400 - 310)}{0.5} = \mathbf{126\ W/m^2} \quad \text{... Ans.}$$

Example 2.3 : *It is necessary to insulate a flat surface so that the rate of heat loss per unit area of this surface does not exceed 450 W/m². The temperature difference across the insulating layers is 400 K (127°C). Evaluate the thickness of insulation if (a) the insulation is made of asbestos cement having thermal conductivity of 0.11 W/(m·K), and (b) the insulation is made of fire clay having thermal conductivity of 0.84 W/(m·K).*

Solution : (a) Area of heat transfer = 1 m² since the heat loss is given per 1 m² of area.

Given :

$Q/A = 450\ W/m^2$, $\Delta T = 400$ K

With $A = 1\ m^2$, $Q = 450$ W

k for asbestos = 0.11 W/(m·K)

The rate of heat loss is given by

$$Q = \frac{k \cdot A\,\Delta T}{x}$$

$$\therefore \quad x = \frac{kA\,\Delta T}{Q}$$

$$x = \frac{0.11 \times 1 \times 400}{450} = 0.098\ m$$

$$= 98\ mm$$

Thickness of asbestos cement insulation = **98 mm** ... **Ans. (a)**

(b) Area of heat transfer = A = 1 m^2

k for fire clay insulation = 0.84 W/(m·K)

$$\Delta T = 400 \text{ K}$$

$$Q = \frac{k \cdot A\,\Delta T}{x}$$

$$x = \frac{0.84 \times 1 \times 400}{450}$$

$$= 0.747 \text{ m}$$

$$= 747 \text{ mm}$$

Thickness of fire clay insulation = **747 mm** ... **Ans. (b)**

Example 2.4 : *A steam pipeline, 150/160 mm in diameter, carries steam. The pipeline is lagged with a layer of heat insulating material [k = 0.08 W/(m·K)] of thickness 100 mm. The temperature drops from 392.8 K (119.8 °C) to 313 K (40 °C) across the insulating surface. Determine the rate of heat loss per 1 m length of pipe line.*

Solution : Consider 1 m of the pipeline.

$$Q = \frac{k \cdot A_m (T_1 - T_2)}{(r_2 - r_1)}$$

r_1 = inside radius of insulation

= 160/2 = 80 mm = 0.08 m

r_2 = outside radius of insulation

= 80 + 100 = 180 mm = 0.18 m

L = length of pipeline = 1 m, since the heat loss is to be calculated per meter of pipe.

A_m = log mean area = $2\pi r_m L$

$$= \frac{2\pi (r_2 - r_1) L}{\ln (r_2/r_1)} = \frac{2\pi (0.18 - 0.08) \times 1}{\ln (0.18/0.08)} = 0.775 \text{ m}^2$$

k = 0.08 W/(m·K)

T_1 = 392.8 K, T_2 = 313 K

The rate of heat loss per unit length of the pipeline is

$$Q = \frac{0.08 \times 0.775 \times (392.8 - 313)}{(0.18 - 0.08)}$$

= **49.5 W/m** ... **Ans.**

Example 2.5 : *A wall is made of brick of thermal conductivity 1.0 W/(m·K), 230 mm thick. It is lined on the inner face with plaster of thermal conductivity 0.4 W/(m·K) and of thickness 10 mm. If a temperature difference of 30 K is maintained between the two faces, what is the heat flow per unit area of wall ?*

Solution : Let the area of heat transfer be 1 m^2.

Thermal resistance of brick = x_1/k_1A

$$R_1 = \frac{0.230}{1.0 \times 1.0} = 0.230 \text{ K/W}$$

$$\text{Thermal resistance of plaster} = R_2 = \frac{x_2}{k_2 A}$$

$$= \frac{0.010}{0.4 \times 1.0} = 0.025 \text{ K/W}$$

The rate of heat flow per 1 m^2 area is

$$Q = \frac{\Delta T}{R} = \frac{\Delta T}{R_1 + R_2}$$

$$= \frac{30}{0.230 + 0.025}$$

$$= \mathbf{117.6 \ W/m^2}$$ ... **Ans.**

Example 2.6 : *A steam pipeline, 150/160 mm in diameter, is covered with a layer of insulating material of thickness 50 mm. The temperature inside the pipeline is 393 K (120 °C) and that of the outside surface of insulation is 313 K (40 °C). Calculate the rate of heat loss per 1 m length of pipeline.*

Data : *k for pipe is 50 W/(m·K) and k for insulating material is 0.08 W/(m·K).*

Solution : Consider 1 m of the pipeline.

$$\text{Thermal resistance offered by the pipe wall} = R_1 = \frac{r_2 - r_1}{k_1 A_{m_1}}$$

where

$$r_1 = 150/2 = 75 \text{ mm} = 0.075 \text{ m}$$

$$r_2 = 160/2 = 80 \text{ mm} = 0.08 \text{ m}$$

$$A_{m_1} = 2\pi r_{m_1} L$$

where A_{m_1} is the log mean area and r_{m_1} is the log mean radius of the steam pipe.

$$A_{m_1} = \frac{2\pi (r_2 - r_1)}{\ln (r_2/r_1)} L$$

$$= \frac{2\pi (0.08 - 0.075)}{\ln (0.08/0.075)} \times 1 = 0.487 \text{ m}^2$$

$$k_1 = 50 \text{ W/(m·K)}$$

$$\therefore \quad R_1 = \frac{0.08 - 0.075}{50 \times 0.487} = 0.000205 \text{ K/W}$$

$$= 2.05 \times 10^{-4} \text{ K/W}$$

$$\text{Thermal resistance offered by the insulation} = R_2 = \frac{r_3 - r_2}{k_2 A_{m_2}} .$$

$$A_{m_2} = 2\pi r_{m_2} L$$

$$= \frac{2\pi (r_3 - r_2) L}{\ln (r_3/r_2)}$$

where $r_3 = r_2 + 50$ mm

$= 80 + 50 = 130$ mm $= 0.13$ m

$$A_{m_2} = \frac{2\pi\,(0.13 - 0.08)}{\ln\,(0.13/0.08)} \times 1$$

$$A_{m_2} = 0.647 \text{ m}^2$$

$$k_2 = 0.08 \text{ W/(m·K)}$$

$$\therefore \quad R_2 = \frac{(0.13 - 0.08)}{0.08 \times 0.647} = 0.966 \text{ K/W}$$

Total thermal resistance $= R = R_1 + R_2$

$= 2.05 \times 10^{-4} + 0.966 = 0.9662$ K/W

The rate of heat loss per 1 m of the pipeline is

$$Q = \frac{\Delta T}{R} = \frac{393 - 313}{0.9662}$$

$= $ **82.8 W/m** ... **Ans.**

Example 2.7 : *A furnace is constructed with 225 mm thick of fire brick, 120 mm of insulating brick and 225 mm of the building brick. The inside temperature is 1200 K (927 °C) and the outside temperature is 330 K (57 °C). Find the heat loss per unit area and the temperature at the junction of the fire brick and insulating brick.*

Data : *k for fire brick = 1.4 W/(m·K)*

k for insulating brick = 0.2 W/(m·K)

k for building brick = 0.7 W/(m·K)

Solution : Let the area of heat transfer be 1 m^2. Therefore, A = 1 m^2.

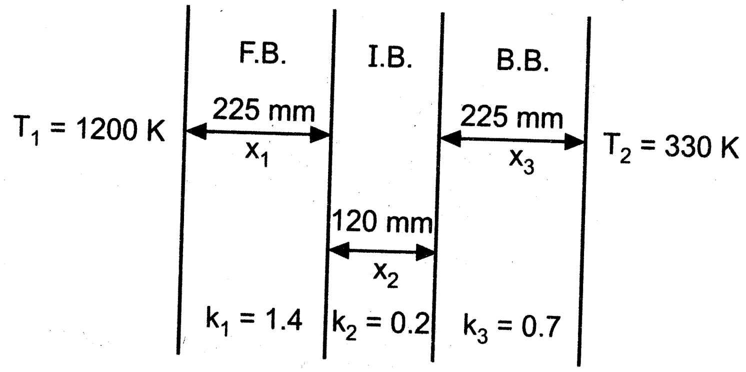

Fig. Ex. 2.7

Let T_1 and T_2 be the temperatures at the fire brick / insulating brick and the insulating brick/building brick junctions respectively.

$$\text{Thermal resistance of the fire brick} = \frac{x_1}{k_1 A}$$

$$R_1 = \frac{0.225}{1.4 \times 1} = 0.1607 \text{ K/W}$$

$$= \frac{550 - 330}{0.0004 + 1.2688 + 2.2385}$$

$$= \mathbf{62.7\ W/m}$$ ... **Ans.**

Example 2.9 : *A wall of 0.5 m thickness is constructed using a material having a thermal conductivity of 1.4 W/(m·K). The wall is insulated with a material having thermal conductivity of 0.35 W/(m·K) so that heat loss per m² is 1500 W. The inner and outer temperatures are 1273 K (1000 °C) and 373 K (100 °C) respectively. Calculate the thickness of insulation required and temperature of the interface between two layers.*

Solution : Let the thickness of insulation required be x_2 metres.

Given : $T_1 = 1273$ K, $T_2 = 373$ K, $k_1 = 1.4$ W/(m·K), $k_2 = 0.35$ W/(m·K), $x_1 = 0.5$ m

The rate of heat transfer per unit area is given by

$$\frac{Q}{A} = \frac{(T_1 - T_2)}{x_1/k_1 + x_2/k_2}$$

$$1500 = \frac{(1273 - 373)}{0.5/1.4 + x_2/0.35}$$

Solving, we get

$$x_2 = 0.085 \text{ m} = 85 \text{ mm}$$

Thickness of insulation required = **85 mm** ... **Ans.**

Let T' be the temperature at the interface.

$$\frac{Q}{A} = T_1 - T'/(x_1/k_1)$$

$$1500 = (1273 - T')/(0.5/1.4)$$

$$T' = \mathbf{737.3\ K\ (464.3\ ^oC)}$$ ... **Ans.**

Example 2.10 : *A cylindrical tube has inner diameter of 20 mm and outer diameter of 30 mm. Find out the rate of heat flow from tube of length 5 m if inner surface is at 373 K (100° C) and outer surface is at 308 K (35° C). Take the thermal conductivity of tube material as 0.291 W/(m·K).*

Solution : Basis : Tube of length 5 metres.

The equation to be used for calculating the rate of heat flow through the tube (cylinder) is

$$Q = \frac{k \cdot 2\pi\, r_m L\, (T_1 - T_2)}{(r_2 - r_1)} \quad \text{... (A)}$$

where, Thermal conductivity = $k = 0.291$ W/(m·K)

Length = $L = 5$ metres

Inner radius = $r_1 = 10$ mm $= 0.01$ m

Outside radius $= r_2 = 15 \text{ mm} = 0.015 \text{ m}$

Inside temperature $= T_1 = 373 \text{ K}$

Outside temperature $= T_2 = 308 \text{ K}$

$$r_m = \text{log mean radius} = \frac{r_2 - r_1}{\ln\left(\frac{r_2}{r_1}\right)}$$

$$= \frac{0.015 - 0.01}{\ln\left(\frac{0.015}{0.01}\right)} = 0.0123 \text{ m}$$

Putting the values of the terms involved in Equation (A), we get

$$Q = \frac{0.291 \times 2\pi\,(0.0123) \times 5\,(373 - 308)}{(0.015 - 0.01)}$$

$$= 1460.8 \text{ W} \equiv \mathbf{1460.8\ J/s}$$ ... **Ans.**

Example 2.11 : *88 mm O.D. pipe is insulated with a 50 mm thickness of an insulation having a mean thermal conductivity of 0.087 W/(m·K) and 30 mm thickness of an insulation, having mean thermal conductivity of 0.064 W/(m·K). If the temperature of the outer surface of the pipe is 623 K (350 °C) and the temperature of the outer surface of insulation is 313 K (40 °C), calculate the heat loss per metre of pipe.*

Solution : Basis : One metre length of pipe.

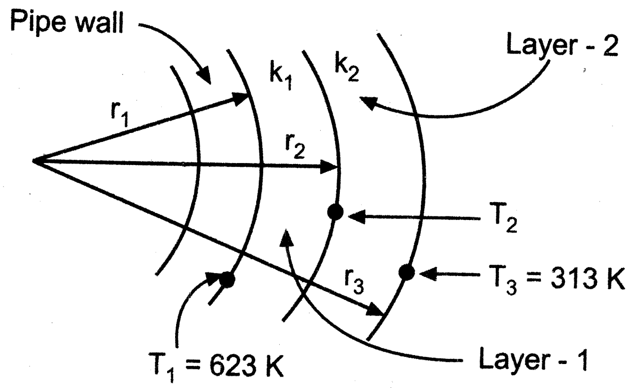

Fig. Ex. 2.11

Refer to Fig Ex. 2.11

$$r_1 = \frac{88}{2} = 44 \text{ mm} = 0.044 \text{ m}$$

$$r_2 = 44 + 50 = 94 \text{ mm} = 0.094 \text{ m}$$

$$r_3 = 44 + 50 + 30 = 124 \text{ mm} = 0.124 \text{ m}$$

Rate of heat flow through a thick-walled cylinder of radii r_1 and r_2 is given by

$$Q = \frac{k_1 (2\pi r_{m_1} L)(T_1 - T_2)}{(r_2 - r_1)}$$

$$\therefore \quad Q = \frac{T_1 - T_2}{\dfrac{r_2 - r_1}{k_1 (2\pi r_{m_1} L)}} \quad \text{... general equation for cylinder.}$$

Similarly, the heat loss through the combined insulation layers is given by

$$Q = \frac{T_1 - T_3}{\dfrac{r_2 - r_1}{k_1 (2\pi r_{m_1} L)} + \dfrac{r_3 - r_2}{k_2 (2\pi r_{m_2} L)}} \quad \text{... (A)}$$

$$[Q_1 = \Delta T_1 / R_1$$

$$Q_2 = \Delta T_2 / R_2$$

$$\Delta T_1 = Q_1 R_1 \text{ and } \Delta T_2 = Q_2 R_2$$

$$\Delta T = \Delta T_1 + \Delta T_2$$

$$\Delta T = Q_1 R_1 + Q_2 R_2$$

But $Q_1 = Q_2 = Q$

$$\therefore \quad \Delta T = Q [R_1 + R_2]$$

$$Q = \frac{\Delta T}{R_1 + R_2} = \frac{T_1 - T_3}{R_1 + R_2}]$$

As $T_1 - T_3 = \Delta T = (T_1 - T_2) + (T_2 - T_3)$

where, ΔT = overall temperature drop

T_1 = temperature at the outer surface of the wall = 623 K

T_3 = temperature at the outer surface of the outer insulation = 313 K

k_1 = thermal conductivity of insulation-1 = 0.087 W/(m·K)

k_2 = thermal conductivity of insulation-2 = 0.064 W/(m·K)

L = Length of pipe = 1 metre

r_{m_1} = log mean radius of insulation layer-1

$$\therefore \quad r_{m_1} = \frac{r_2 - r_1}{\ln\left(\dfrac{r_2}{r_1}\right)}$$

$$= \frac{(0.094 - 0.044)}{\ln\left(\dfrac{0.094}{0.044}\right)} = 0.066 \text{ m}$$

r_{m_2} = log mean radius of insulation layer - 2

$$r_{m_2} = \frac{r_3 - r_2}{\ln\left(\frac{r_3}{r_2}\right)} = \frac{0.124 - 0.094}{\ln\left(\frac{0.124}{0.094}\right)} = 0.1083 \text{ m}$$

Substituting the values of all terms involved in Equation (A), we get

The heat loss per metre of pipe is

$$Q = \frac{(623 - 313)}{\left(\frac{0.05}{0.087 \times 2\pi \times 0.066 \times 1}\right) + \left(\frac{0.03}{0.064 \times 2\pi \times 0.1083}\right)}$$

$$Q = \mathbf{149.4\ W/m}$$ ... **Ans.**

Example 2.12 : *A furnace is constructed with 229 mm thick of fire brick, 115 mm of insulation brick and again 229 mm of building brick. The inside temperature is 1223 K (950 °C) and the temperature at the outermost wall is 323 K (50°C). The thermal conductivities of fire brick, insulating brick and building brick are 6.05, 0.581 and 2.33 W/(m·K). Find the heat lost per unit area and temperatures at the interfaces.*

Solution :

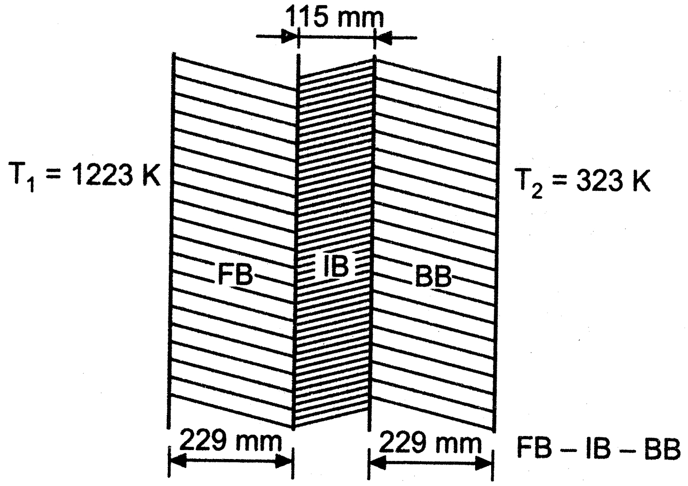

Fig. Ex. 2.12

Assume :

Heat transfer area = A = 1 m²

Given : x_1 = thickness of fire brick = $\frac{229}{1000}$ = 0.229 m

x_2 = thickness of insulating brick = $\frac{115}{1000}$ = 0.115 m

x_3 = thickness of building brick = $\frac{229}{1000}$ = 0.229 m

k_1 = thermal conductivity of fire brick = 6.05 W/(m·K)

k_2 = thermal conductivity of insulating brick = 0.581 W/(m·K)

k_3 = thermal conductivity of building brick = 2.33 W/(m·K)

T_A = temperature at the interface between fire brick and insulating brick (K)

T_B = temperature at the interface between insulating brick and building brick (K)

T_1 = 1223 K, inside temperature

T_2 = 323 K, outside temperature

Overall temperature drop is

$$\Delta T = 1223 - 323 = 900 \text{ K}$$

Let us calculate Q (heat loss/m^2).

The rate of heat lost per unit area is given by

$$Q = \frac{\Delta T}{R_1 + R_2 + R_3} = \frac{900}{\left(\frac{x_1}{k_1 A}\right) + \left(\frac{x_2}{k_2 A}\right) + \left(\frac{x_3}{k_3 A}\right)}$$

$$= \frac{900}{\frac{0.229}{6.05 \times 1} + \frac{0.115}{0.581 \times 1} + \frac{0.229}{2.33 \times 1}} = \mathbf{2694\ W/m^2}$$

Let us calculate T_A.

The rate of heat transfer through the fire brick layer is given by

$$Q_1 = \frac{T_1 - T_A}{\frac{x_1}{k_1 A}}$$

But under steady state heat transfer conditions, $Q_1 = Q$. Therefore,

$$Q_1 = Q = 2694 = \frac{1223 - T_A}{\frac{0.229}{6.05 \times 1}}$$

$$\therefore \quad 1223 - T_A = 102$$

$$\therefore \quad T_A = 1121 \text{ K } (848^\circ \text{C})$$

Let us calculate T_B.

For steady state heat transfer,

$$Q_3 = Q = \frac{T_B - T_2}{x_3/k_3 A} = \frac{T_B - 323}{x_3/k_3 A}$$

$$2694 = \frac{T_B - 323}{\frac{0.229}{2.33 \times 1}}$$

$$\therefore \quad T_B = 587.8 \text{ K } (314.8^\circ \text{C})$$

Q based on 1 m^2 heat transfer surface = 2694 W