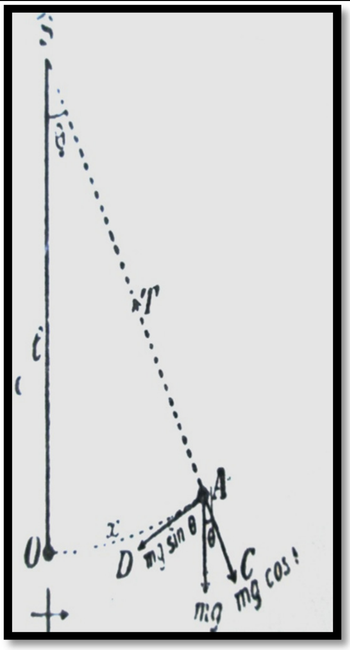
Acceleration due to gravity: All bodies, irrespective of their mass or nature, falling freely in vacuum, will have the same acceleration at a given place. This acceleration is called acceleration due to gravity.
It is denoted by 'g' and its value differs from place to place. It is greatest at the poles and the least at the equator. It is generally taken to be 980 cm/s^2 in CGS system and 9.8 m/s^2 in MKS system on the surface of the Earth. It is determined with the help of a simple pendulum or a compound pendulum.

The simple pendulum



A simple pendulum consists of a heavy spherical bob (ideally may be treated as point mass) suspended from a fixed point by an inextensible, weightless string.

In Figure, S is the point of suspension, O is the centre of the bob, 'x' is the displacement of the bob from O and $'l'$ is the length of the pendulum. Let OS is the position of bob at rest and SA is the displaced position of the bob through angle 'θ'.
The pendulum in A position is subjected to two forces (i) its weight acting vertically downward and (ii) the tension of the string along AS towards its point of suspension.
The force mg due to its weight can be resolved into two components (i) mg cos θ acting along SA and
(ii) mg sin θ at right angles to SA.
As there is no motion along SA the force mg cos θ balances the tension in the string. Hence the only force which acts on the bob is mg sin θ and acts towards the mean position of the bob O.
Acceleration of the bob =mg sin θ/m = g sin θ.
[∵ acceleration = force / mass]
This acceleration is directed toward AO or towards the mean position of the bob O. If θ is very small we have sin θ≈θ

∴ Acceleration of the bob =g θ $[\because \theta = \frac{\text{arc}}{\text{radius}} = \frac{\text{OA}}{\text{SO}} = \frac{x}{l}]$

∴ Acceleration of the bob $= \frac{gx}{l}$

Thus, the bob's acceleration is proportional to its displacement from its mean position (x) and always directed towards its mean position i. e. it executes simple harmonic motion. The periodic time (T) of the pendulum is

$$\therefore T = 2\pi\sqrt{\frac{1}{g/l}} = 2\pi\sqrt{\frac{l}{g}} . \qquad \text{OR} \qquad g = 4\pi^2\left(\frac{l}{T^2}\right).$$

Hence, by substituting periodic time (T) and length of the simple pendulum (l) in above equation we can determine 'g'.

Drawbacks of simple pendulum

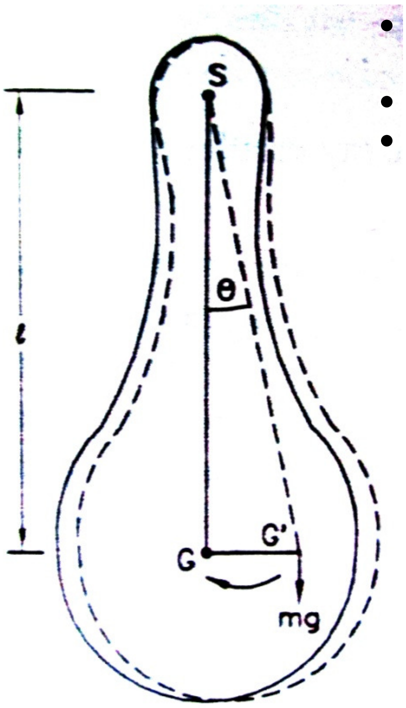
This method of finding the value of g has the following drawbacks:

- Simple pendulum is only ideal concept because point mass bob and weightless string is practically not possible.
- We have not considered effect of the air resistance and buoyancy.
- The expression for the time period is true only if displacement is infinitely small.
- The motion of the bob is not a motion of translation. The rotary motion of the bob should also consider in the formula.
- The bob is not rigidly connected with the string and has some relative motion with respect to thread

when approaching the limits of the string.

Compound OR Physical Pendulum

A rigid body capable for oscillating in a vertical plane about a fixed horizontal axis through pivot is called compound or physical pendulum.



- The motion of compound pendulum is simple harmonic and its periodic time
- is given
- by

$$T = 2\pi\sqrt{\frac{I}{mgl}} \qquad (1)$$

Where '**I**' is the moment of inertia of a rigid body about the axis through point
of suspension and 'm' is the mass of rigid body, 'l' be the pendulum length
or the distance between the point of suspension and centre of gravity (CG).
Let S be the point of suspension of a compound pendulum. The center of gravity (CG) of a rigid body lies vertically below the point of suspension in equilibrium position. Let θ be the angle between rest and deflected position.

The couple acting on a rigid body is mg $l$sin θ. Due to this couple the rigid body will tends to come in its original position.

If $\frac{d\vec{\omega}}{dt}$ is the angular acceleration of the rigid body then the toque or couple is equal to $I\frac{d\omega}{at}$.

$$\therefore \vec{\tau} = I\,\vec{\alpha} = -\frac{d\vec{\omega}}{dt} \qquad (2)$$

$$\text{But} \therefore \vec{\tau} = \vec{r} \times \vec{F} = \vec{l} \times m\,\vec{g} = -\,m\,g\,l\,\sin\theta \qquad (3)$$

(∵restoring force)

Comparing equations (2) and (3), we get

$$I\,\vec{\alpha} = \frac{d\vec{\omega}}{dt} = -m\,g\,l\,\sin\theta \quad \text{OR} \quad \therefore \frac{d\vec{\omega}}{dt} = -\frac{m\,g\,l}{I}\sin\theta \quad \text{OR} \quad \therefore \frac{d\vec{\omega}}{dt} = -\mu\,\theta \quad \left[\because \mu = \frac{m\,g\,l}{I}\right]$$

$$\therefore \frac{d\vec{\omega}}{dt} + \mu\,\theta = 0 \quad (4)$$

This is the equation of motion of compound pendulum. Equation (4) shows that angular acceleration is proportional to the angular displacement. This angular acceleration executes simple harmonic motion. Hence, the periodic time is

$$T = 2\pi\sqrt{\frac{1}{\mu}} = 2\pi\sqrt{\frac{1}{mgl/I}} \qquad \text{OR} \qquad T = 2\pi\sqrt{\frac{I}{mgl}} \qquad (5)$$

If I_o be the moment of inertia of rigid body about the axis through CG and parallel to the point of suspension, then by using parallel axis theorem

$$I = I_o + ml^2 \qquad (6)$$

If k is the radius of gyration then $I_0 = mk^2$. Substituting this value in equation (6), we get

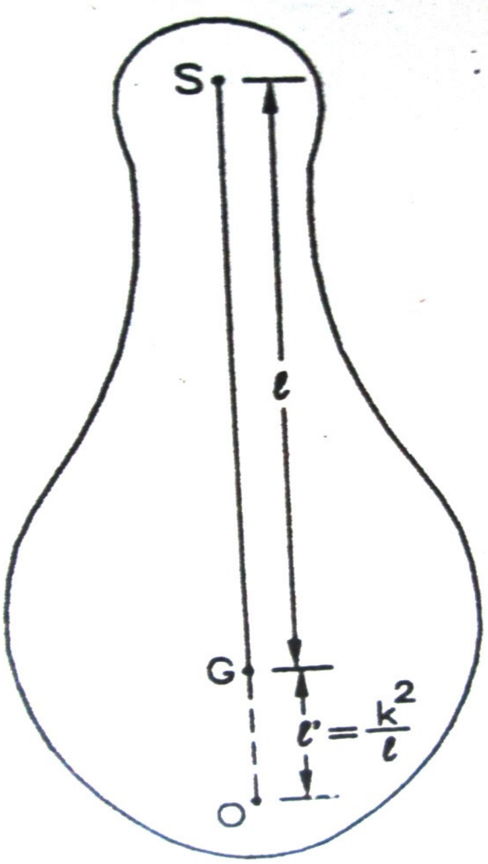
$$I = mk^2 + ml^2 \qquad (7)$$

From equations (5) and (7), we get

$$T=2\pi\sqrt{\frac{m\,k^2+m\,l^2}{m\,g\,l}} \quad \text{OR} \quad T=2\pi\sqrt{\frac{k^2+l^2}{g\,l}} \quad \text{OR} \quad T=2\pi\sqrt{\frac{k^2/l+l}{g}} \qquad (8)$$

The period of oscillation is same as the simple pendulum of length $k^2/l+l$. This length is called the length of equivalent simple pendulum or reduced length of pendulum. It is denoted by L. Since k^2 is never zero. Hence equivalent simple pendulum length is always greater than l.

Centre of Oscillation



Let O be the point on the other side of the CG at a distance k^2/l. This point is called centre of oscillation. The axis passing through this point and parallel to the point of suspension is called axis of oscillation.

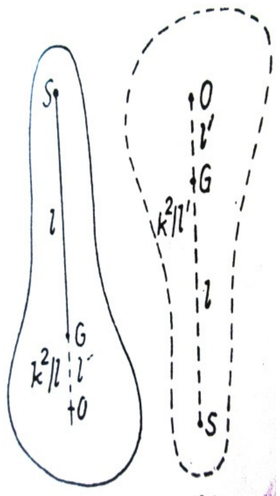
From Figure, SO= SG+GO=$l+k^2/l$.

If L=l + l' , the periodic time is $T=2\pi\sqrt{\frac{k^2/l+l}{g}}$ OR $T = 2\pi\sqrt{\frac{L}{g}}$

Thus the point of oscillation O lies vertically at a distance L from the point of suspension. The distance between these two points is called the length of equivalent simple pendulum.

Interchangeability of centers of suspension and oscillation

If the pendulum be inverted and suspended about the axis of oscillation through O, as shown in Figure 4. Its time period will be obviously given by $T=2\pi\sqrt{\frac{k^2+l'^2}{g\,l'}}$. And, since $k^2/l = l'$ we have $k^2 = l\,l'$ so that the expression for the time period t becomes $T=2\pi\sqrt{\frac{l\,l'/l+l}{g}}$ OR $T=2\pi\sqrt{\frac{l'+l}{g}}$



The time period about point of oscillation is same as the axis of suspension through S. Thus the centre of oscillation and oscillation are interchangeable.

Centre of percussion

Figure shows a section of a rigid body of mass 'm' by a vertical plane passing though its centre of gravity 'G' and point of suspension 'S'. Let F be the force applied at 'O' in a direction perpendicular to the vertical line

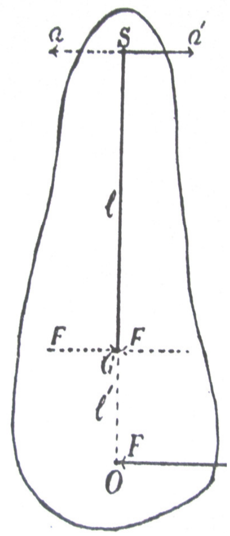


Fig 5

passing through SGO.

This force $\vec{F}$ at G tends to produce linear acceleration in the body i.e. $\vec{F} = m \times \vec{a}$.
The acceleration $\vec{a} = \vec{F}/m$ is in the direction of force i.e. from right to left.
This force $\vec{F}$ also produces angular acceleration about G. If 'I' be the moment of inertia about the axis through CG then

$$I\,\vec{\alpha} = \vec{F} \times \vec{l'} \quad \therefore \vec{\alpha} = \frac{\vec{F} \times \vec{l'}}{I} \quad \text{OR} \quad \therefore \vec{\alpha} = \frac{\vec{F} \times \vec{l'}}{mk^2} \quad (1),$$

where $I = mk^2$, k is radius of gyration.
But,
linear acceleration=angular acceleration X distance from the axis.

The linear acceleration is produced by this couple is given by

$$\vec{a'} = \vec{l} \times \vec{\alpha} = \vec{l} \times \vec{F} \times \frac{\vec{l}}{mk^2} \qquad (2)$$

The direction of $\vec{a'}$ is from left to right i.e. in opposite to that of $\vec{a}$. Hence, force $\vec{F}$' applied at O may produce no effect at S. i.e. $\vec{a} = \vec{a'}$

$$\therefore \frac{\vec{F}}{m} = l \times \vec{F} \times \frac{\vec{l'}}{mk^2} \quad \text{OR} \quad l' = k^2/l. \qquad (3)$$

This is the distance of the point O from the CG of the body. The point O is the centre of oscillation. This point is called the centre of percussion with respect to S.

Thus if a body be struck at the centre of percussion, or the centre of oscillation, in a direction perpendicular to its axis of suspension, it does not move bodily as a whole, at its point of suspension, but simply turns about the axis passing through it.

Other points, collinear with CG about which the time period is the same

The time period for compound pendulum is $t = 2\pi\sqrt{\frac{k^2 + l^2}{g\,l}}$. (1)
Squaring above equation, we get

$$t^2 = 4\pi^2 \frac{k^2 + l^2}{g\,l} \quad \text{OR} \quad g\,lt^2 = 4\pi^2(k^2 + l^2) \quad \text{OR} \quad \therefore \frac{g\,l\,t^2}{4\pi^2} = k^2 + l^2 \text{ OR}$$

$$\therefore l^2 - \frac{g\,t^2}{4\pi^2}\,l + k^2 = 0 \quad (2)$$

This is a quadratic equation in l. We know that, the solution of quadratic equation

$$ax^2 + bx + c = 0 \quad (3) \quad \text{is} \quad \Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \qquad (4)$$

Comparing the coefficients of equation (2) and (3) we have a=1, $b = -\frac{g\,t^2}{4\pi^2}$ & $c = k^2$

The solution of equation (2) is

$$l = \frac{\frac{g\,t^2}{4\,\pi^2} \pm \sqrt{(\frac{g\,t^2}{4\,\pi^2})^2 - 4k^2}}{2} \quad (5)$$ which has two solution

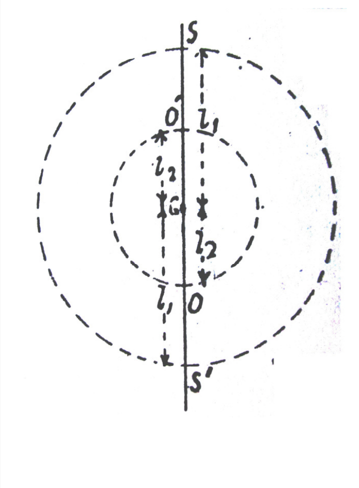
$$l_1 = \frac{g\,t^2}{8\,\pi^2} + \sqrt{(\frac{g\,t^2}{8\,\pi^2})^2 - k^2} \quad (6) \text{ and } \quad l_2 = \frac{g\,t^2}{8\,\pi^2} - \sqrt{(\frac{g\,t^2}{8\,\pi^2})^2 - k^2} \quad (7)$$

Adding equations (6) and (7), we get

$$l_1 + l_2 = \frac{2\,g\,t^2}{8\,\pi^2} \quad \text{OR} \quad l_1 + l_2 = \frac{g\,t^2}{4\,\pi^2} \quad (8)$$

Multiplying equations (6) and (7)

$$l_1 l_2 = \left[\frac{g\,t^2}{8\,\pi^2}\right]^2 - \left[\left(\frac{g\,t^2}{8\,\pi^2}\right)^2 - k^2\right] \quad \therefore\ l_1 l_2 = k^2 \quad (9)$$



Equation (8) and (9) shows that l_1 and l_2 are positive. Thus, there are two points of the compound pendulum for the same periodic time at a distance l_1 and of the l_2 (l_2=k^2/l_1) about the centre of gravity, we get four points on the axis passing through point of suspension and centre of gravity. The time period is same for these four points. Hence these points on the axis are said to be collinear about the CG which has same periodic time.

In this Figure O is the centre of oscillation for S and O' is centre of oscillation for S'.

$$\therefore\ SO = S'O' = l_1 + l_2 = l_1 + k^2/l_1 = L \quad (10)$$

From equation (8) and (10) $l_1+l_2 = L = \frac{g\,t^2}{4\,\pi^2}$ (11)

Using equation (11) we can determine the value of g.

Conditions for maximum and minimum time period

We know that, the time period for compound pendulum is $t = 2\,\pi\sqrt{\frac{k^2 + l^2}{g\,l}}$ (1)

Squaring above equation, we get $t^2 = 4\,\pi^2(\frac{k^2+l^2}{g\,l})$ OR $t^2 = \frac{4\,\pi^2}{g}(\frac{k^2}{l} + l)$

Differentiating above equation with respect to l, we get

$$2\,t\,\frac{dt}{dl} = \frac{4\,\pi^2}{g}(-\frac{k^2}{l^2} + 1)$$

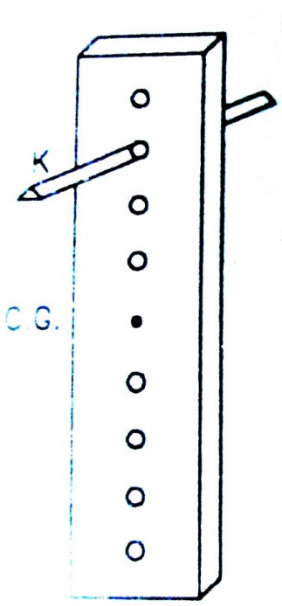
The time period t will be minimum when $\frac{dt}{dl}$=0, for $k^2 = l^2$ or l=k. Hence if the distance between point of suspension and CG is equal to the radius of gyration t will be minimum.

When l=0, t will be maximum i.e. when the axis passing through the CG, the time period will be maximum.

Bar Pendulum

The simplest form of compound is bar pendulum.

A uniform rectangular metal bar (AB) having holes drilled along its length symmetrically on both the side of its centre of gravity which is suspended from horizontal knife-edge and oscillate in a vertical plane is called bar pendulum.



Determination of g using Bar pendulum

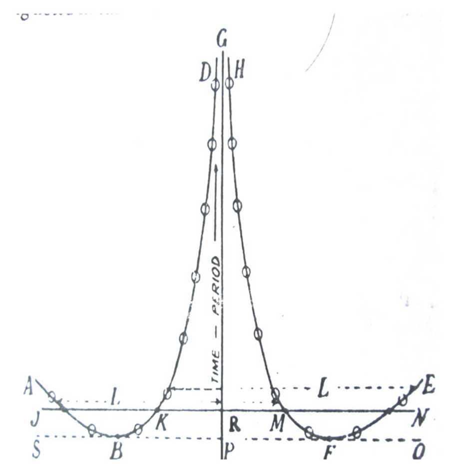
A bar pendulum is a particular case of a compound pendulum. The time period is determined by fixing the knife edge in each hole. The distance of each hole from the centre of gravity is measured. A graph is drawn between the distance from the CG along the X-axis and the corresponding time period along the y-axis. The graph is as shown in Figure. In Figure the time period decreases at first, acquire a minimum value and then increases as the CG of the bar is approached, finally becoming infinite at the CG. Fig 7

Now, if a horizontal line JN be drawn, it cuts the two curves at points J, K, M and N about which the time period is same. Therefore, JM=KN=L is the length of equivalent simple pendulum.

The points J and M (as K and N) lie on opposite sides of the CG. They corresponds the pendulum respectively. The distance between them thus giving directly the length of the equivalent simple pendulum L.
Once the value of the L is determined the values g is calculated with the help of expression for T i.e. $T = 2\pi\sqrt{\frac{L}{g}}$.

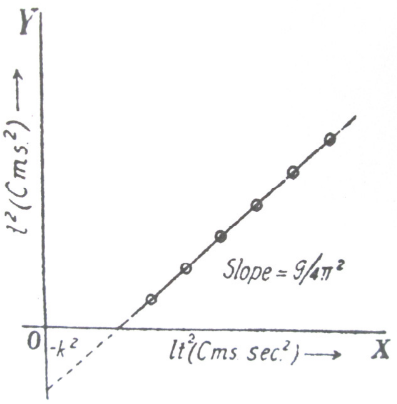
The radius of gyration can be also calculated by measuring the distance M_1 and M_2 as K= BF/2.

Fig 8



The graph as shown in Figure is drawn as a most probable curve as such the value of L and hence the value of g cannot be measured accurately. The improved method of calculating g was suggested by Ferguson.

We know that the relation



$$k^2 + l^2 = \frac{g\,l\,t^2}{4\,\pi^2} \quad \text{OR} \quad \therefore l^2 = \frac{g\,t^2}{4\,\pi^2}\,l - k^2$$

This is the equation of straight line. Hence we plot the graph between lt^2 along the x-axis and l^2 along y-axis gives a straight line as shown in the Figure.

The slope of the line must be equal to $\frac{g}{4\,\pi^2}$ and its intercept is equal to $-k^2$.

The value of g and k can be obtained much more accurately.

Determination of k

If we draw the tangential line SQ, touching the two curves at the points B and F respectively, then at B and F the centre of suspension and oscillation of minimum time periods

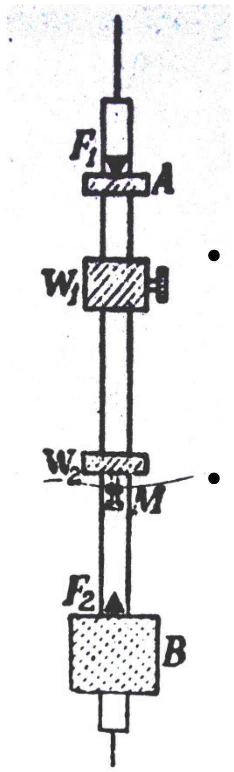
- We know that $L=l_1+l_2$ is the equivalent length of the simple pendulum. In the graph

$JR=NR=l_1$ and $KR=MR=l_2$ $l_1+l_2=JR+MR=JM$ or $l_1+l_2=NR+KR=KN$

The value of g can be determined from $T=2\,\pi\sqrt{\frac{k^2/l_1+l_1}{g}}$

Comparing equations, we get $k^2/l_1 = l_2$ OR $k^2 = l_1\,l_2$

$\therefore k = \sqrt{l_1\,l_2}$ *OR* $\therefore k = \sqrt{JR \times KR}$ OR $\therefore k = \sqrt{NR \times MR}$



Using above equation the radius of gyration k of the bar pendulum can be easily determined.

Kater's reversible pendulum

It consists of a long rod AB having a fixed heavy bob B and also two fixed knife edge F_1 and F_2 which are adjustable and mutually facing at the two ends of the rod.

- The pendulum can be suspended from either side. Two weights W_1 and W_2 which can be made to slide along the length of the bar are clamped in the position desired. The position of the CG is changed by the adjustment of the weights A and B, their position is so chosen that the CG always lies in between the two knife edges. The weight W_2 has an attached micrometer screw arrangement for its finer adjustment of its position.
- The fact that centre of suspension and oscillation can be interchanged is used in determining the equivalent simple pendulum lengths, thus g, from a Kater's pendulum. The time period of a pendulum about a horizontal axis at a distance l_1 from its centre of gravity T_1 is given by

$$T = 2\,\pi\sqrt{\frac{K^2+l_1^2}{l_1 g}} \quad \text{Or} \quad l_1 T_1^2 = \frac{4\,\pi^2}{g}\left(K^2 + l_1^2\right) \qquad (1)$$

-

Now if we reverse the pendulum and make it to vibrate about another parallel horizontal axis a distance l_2 from its centre of gravity, the time period T_2 is given by

$$l_2 T_2^2 = \frac{4\,\pi^2}{g}(\ K^2 + l_2^2) \qquad (2)$$

If two axes are such that they are reciprocal axes of suspension and oscillation then T_1=T_2.
Let us substitute T_1=T_2=T in equations (1) and (2), we have

$$(l_1 - l_2)T^2 = \frac{4\,\pi^2}{g}(\ l_1^2 - l_2^2) \ \text{ if } (l_1 \neq l_2) \text{ then } \ T_2 = \frac{4\,\pi^2}{g}(\,l_1 + l_2\,) \qquad (3)$$

Thus the distance between the two points on opposite sides of the CG and collinear with it but situated at unequal distances from it gives the length of the equivalent simple pendulum (L). Once the value of L is known the g can be calculated from the relation as T can be determined i.e. $g = 4\,\pi^2(\frac{L}{T^2})$.

Questions Bank

	Sort Questions
1	What is simple pendulum?
2	Discuss drawbacks of a simple pendulum.
3	What is a compound pendulum?
4	Distinguish between simple pendulum and compound pendulum. OR Give advantages of compound pendulum over simple pendulum in determining the value of g.
5	Prove that the periodic time of a compound pendulum will be minimum when the length of the pendulum is equal to it's radius of gyration about a horizontal axis passing through its centre of gravity. OR State the condition for minimum time period of a compound pendulum.
6	Prove that the periodic time of a compound pendulum will be maximum when the axis of rotation passing through the CG. OR State the condition for maximum time period of a compound pendulum.
7	Define centre of suspension and centre of oscillation
	Long Questions
1	What is simple pendulum? Derive an expression for the periodic time of a simple pendulum. Discuss drawbacks of a simple pendulum.
2	What is a compound pendulum? Deduce an expression for its periodic time.
3	Prove that there are four points collinear with the centre of gravity of a compound pendulum about which its times of oscillations are equal, hence obtain the length of an equivalent simple pendulum.
4	Prove that the periodic time of a compound pendulum is minimum when the length of the pendulum is equal to it's radius of gyration about a horizontal axis passing through its centre of gravity.
5	Prove that the periodic time of a compound pendulum is maximum when the axis of rotation passing through the CG.
6	Describe Kater's reversible pendulum. Obtain an expression for acceleration due to gravity in terms of two nearly equal periods of oscillation about the two parallel knife-edges.
7	If the period of Kater's pendulum in the erect and inverted positions is equal, prove that the distance between the knife edges equal to the length of simple equivalent pendulum.
8	In case of compound pendulum, show that centers of suspension and oscillation are reversible (or interchangeable).
9	Draw diagram of bar pendulum and explain how to determine g using bar pendulum.
10	Draw diagram of Kater's pendulum. Stating the meaning of each symbol write the equation for the 'g' using Kater's pendulum.

MCQ

1	If we increase the length of simple pendulum its time period will ___________.			
	(a)	**Increase**	(b)	Decrease
	(c)	Remain same	(d)	becomes infinite
2	A simple pendulum that behaves as a seconds pendulum on the earth. If it is taken to moon, where gravitational acceleration is one sixth that on the earth. Its time period will become			
	(a)	4 second	(b)	3.5 second
	(c)	12 second	**(d)**	**4.9 second**
3	The period of simple pendulum is doubled when			
	(a)	Its length is doubled	(b)	Its length is halved
	(c)	**The length is made four times**	(d)	Mass of the bob is doubled
4	The acceleration due to gravity changes from 9.8 m/s^2 to 9.5 m/s^2. To keep the period of pendulum constant, its length must changes by			
	(a)	3 m	(b)	0.3 m
	(c)	0.3 cm	**(d)**	**3 cm**
5	The time period of simple pendulum having infinite length is			
	(a)	Zero	(b)	1
	(c)	**Infinite**	(d)	None of these
6	If we increase the mass of the bob of simple pendulum its time period will ___________.			
	(a)	Increase	**(b)**	**Remain same**
	(c)	Decrease	(d)	becomes infinite
7	A simple pendulum is performing S.H.M with period T. If its length is doubled. The new time period will be ___________.			
	(a)	2 T	(b)	2.5 T
	(c)	0.5 T	**(d)**	**1.41 T**
8	There are _____________ points collinear with the centre of gravity of a compound pendulum about which its times of oscillations are equal.			
	(a)	2	(b)	6
	(c)	**4**	(d)	8
9	The periodic time of a compound pendulum will be ______________ when the length of the pendulum is equal to it's radius of gyration about a horizontal axis passing through its centre of gravity.			
	(a)	Remain same	**(b)**	**Minimum**
	(c)	None of the these	(d)	Maximum
10	The periodic time of a compound pendulum will be ___________when the axis of rotation passing through the CG.			
	(a)	Remain same	(b)	Minimum
	(c)	None of the these	**(d)**	**Maximum**
11	If the length of simple pendulum is increased by 44% then what is the change in the time period of the pendulum?			
	(a)	22 %	(b)	44 %
	(c)	20 %	(d)	33 %

Correct Answers of MCQs

1	(a)	**2**	(d)	**3**	(c)	**4**	(d)
5	(c)	**6**	(b)	**7**	(d)	**8**	(c)

Fig 10

9	(b)	**10**	(d)	**11**	(c)	**12**	