PHYSICS COURSE : US01CPHY02

UNIT – 2 : BRIDGES AND THEIR APPLICATIONS

DC BRIDGES

INTRODUCTION

Bridges are used to measure the values of the electronic components. For example a Wheatstone Bridge is used to measure the unknown resistance of a resistor. However they are also used to measure the unknown inductance, capacitance, admittance, conductance or any of the impedance parameters.

Besides this bridge circuits are also used in the precision measurements in some circuits and for the interfacing of transducers. Actually nowadays fully automatic bridges which electronically null a bridge to make precision measurements are also used.

WHEATSTONE BRIDGE : BASIC OPERATION

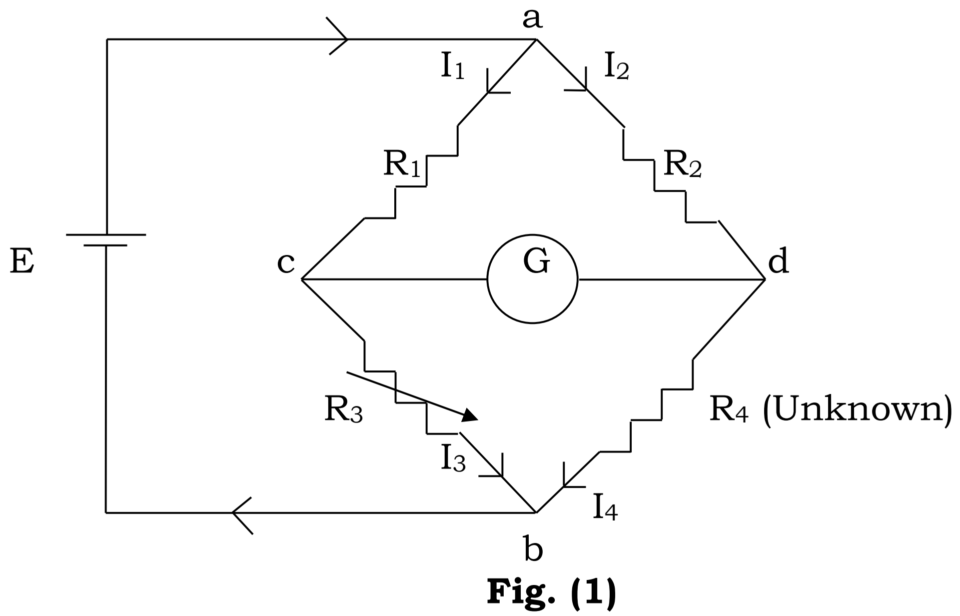

Fig. (1)

• The circuit diagram of Wheatstone Bridge is shown in Fig. (1). The four arms of the bridge ac, ad, cb and db contains the four resistors R_1, R_2, R_3 and R_4 respectively. G is a galvanometer or the null detector. E is the source of EMF.

• I_1, I_2, I_3 and I_4 are the currents through the resistors R_1, R_2, R_3 and R_4, respectively.

• When the current through galvanometer is zero, at that time terminals c and d are said to be at same potential with respect to point a i.e.,

$$E_{ac} = E_{ad} \qquad (1)$$

• Hence the currents $I_1 = I_3$ and $I_2 = I_4$. This is called the balance of the bridge. And for this condition, we can write,

$$I_1R_1 = I_2R_2 \qquad (2)$$

Where

$$I_1 = I_3 = \frac{E}{R_1+R_3} \qquad (3)$$

and

$$I_2 = I_4 = \frac{E}{R_2+R_4} \qquad (4)$$

• Substituting the values of I_1 and I_2 from equations (3) and (4) into (2), we get

$$\frac{E}{R_1+R_3}R_1 = \frac{E}{R_2+R_4}R_2$$

$$\frac{R_1}{R_1+R_3} = \frac{R_2}{R_2+R_4}$$

therefore, $R_1(R_2+R_4) = R_2(R_1+R_3)$

$$R_1R_4 = R_2R_3 \qquad (5)$$

• Equation (5) is called the balance equation(condition) of the bridge. Here, if R_4 is an unknown resistor, then its resistance R_x can be measured using the equation

$$R_1R_x = R_2R_3$$

$$R_x = \frac{R_2 R_3}{R_1} \tag{6}$$

• Here resistor R_3 is called the standard arm, whereas R_2 and R_1 are called the ratio arms.

MEASUREMENT ERRORS (LIMITATIONS OF THE WHEATSTONE BRIDGE CIRCUIT *OR* SOURCES OF ERRORS IN WHEATSTONE BRIDGE MEASURREMENTS)

• The major sources of errors in the Wheatstone bridge measurements are as follows :

(1) Insufficient sensitivity of the null detector (galvanometer).

(2) Heating effects of currents through the arm resistors : The currents flowing thourgh the resistors produce heating effects in them and changes the resistance values. This affects the bridge balance. It leads to the I^2R losses in resistors. If an excess amount of current flows, the resistance values change permanently and this affects the subsequent measurements. This error becomes substantial particularly when low resistance values are to be measured. To avoid this, the current must be limited to a safe value.

(3) Thermal EMF in the bridge circuit or the galvanometer circuit can also cause problems when low-value resistors are being measured. This limitation can be overcome using high sensitivity galvanometer having copper coil and copper suspension system in it.

(4) Leads and Contacts : the resistance drops in the leads (wires) and contacts which are used in the external

bridge circuit affects the balance of bridge. This error can be minimized using a Kelvin bridge circuit.

THEVENIN EQUIVALENT CIRCUIT (OF WHEATSTONE BRIDGE)

• With what accuracy a Wheatston bridge can be balanced depends upon the sensitivity of galvanometer.

• The amount of current per unit deflection is called the sensitivity of galvanometer.

• The balance of bridge is also affected by the internal resistance of galvanometer.

• The suitability of galvanometer for a particular bridge circuit can be known by using Thevenin equivalent circuit of that bridge.

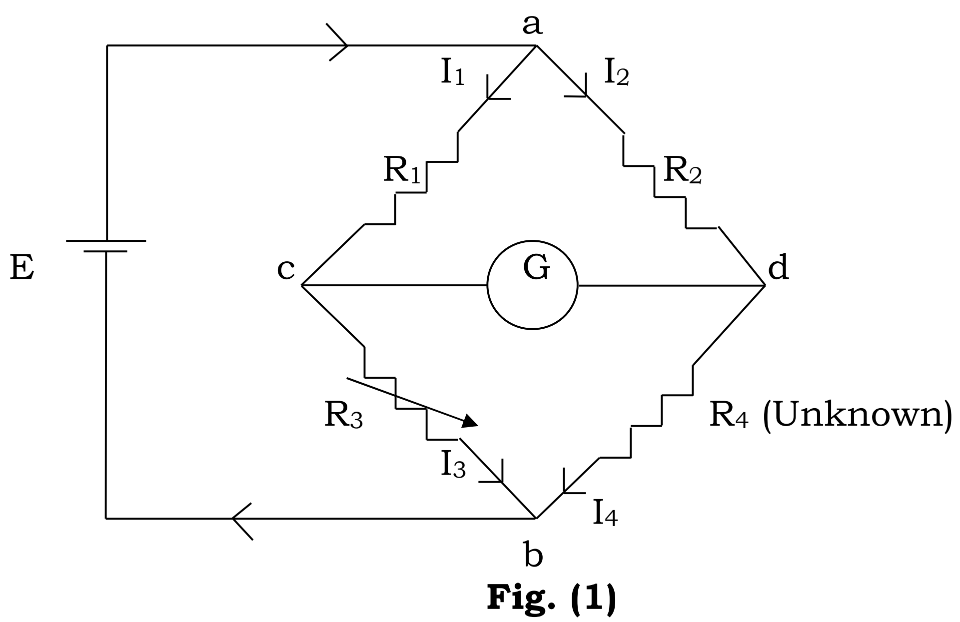

Fig. (1)

• Let us consider a Wheatston bridge circuit as shown in Fig. (1).
• When the bridge is balanced, points c and d are at the same potential with respect to point. Therefore, we can write

$$E_{cd} = E_{ac} - E_{ad} = I_1R_1 - I_2R_2 \qquad (1)$$

• Also we can write currents I_1 and I_2 as

$$I_1 = I_3 = \frac{E}{R_1+R_3} \qquad (2)$$

and $$I_2 = I_4 = \frac{E}{R_2+R_4} \qquad (3)$$

• Then, substituting values of I_1 and I_2 from equations (2) and (3) into equation (1), we get

$$E_{cd} = E\left(\frac{R_1}{(R_1+R_3)} - \frac{R_2}{(R_2+R_4)}\right) \qquad (4)$$

• In the circuit of Fig. (1) we can find the current through galvanometer by obtaining the Thevenin equivalent circuit of the bridge looking into the terminals c and d. This circuit is shown in Fig. (2).

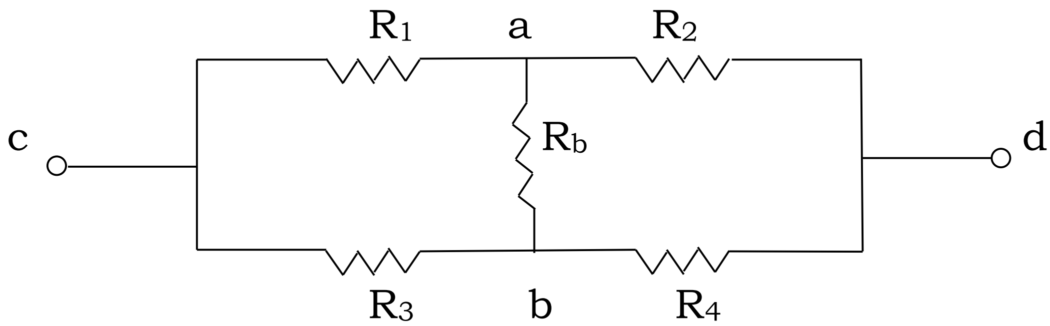

Fig. (2)

• The Thevenin Resistance , looking into the terminals c and d in Fig. (2) then becomes

$$R_{TH} = \left(\frac{R_1R_3}{(R_1+R_3)} + \frac{R_2R_4}{(R_2+R_4)}\right) \qquad (5)$$

• The Thevenin equivalent voltage source E_{TH} is given by

$$E_{TH} = E_{cd}$$

• Now the Thevenin equivalent circuit can be shown as shown in Fig. (3) below.

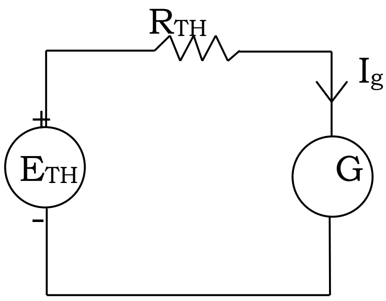

Fig. (3)

• Then the current through galvanometer is given by

$$I_g = \frac{E_{TH}}{R_{TH}+R_g} \tag{6}$$

Where R_g is the resistance of galvanometer.

LIMITATIONS OF WHEATSTONE BRIDGE

• The Wheatstone bridge is limited to the measurement of resistances ranging from a few ohms to several megohms.

• The upper limit is set by the reduction in sensitivity of the bridge to unbalance condition, which is produced due to high resistance values of Thevenin equivalent resistance. This reduces the galvanometer current.

• The lower limit is set by the resistance of the connecting leads(wires/cables) and the contact resistances at the binding points. These resistances makes the measurements of low resistances difficult.

KELVIN BRIDGE : EFFECTS OF CONNECTING LEADS

• The Kelvin bridge is the modification of Wheatstone bridge. It measures the low resistance values with more accuracy.
• Fig. (1) below shows the circuit diagram for the Kelvin bridge circuit.

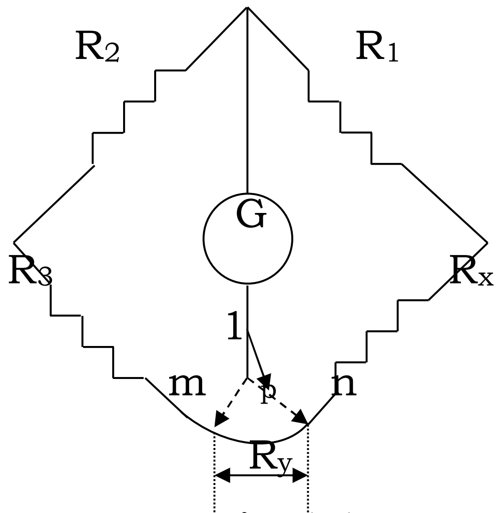

Fig. (1)

•As shown in the Fig. (1) above R_y is the rresistance of the connecting lead from R_3 to R_x. Points m and n shows the two possible connections for galvanometer.
• When connection is made along 1n, R_y is added to R_3 and when connection is made along 1m, R_y is added to R_x. However, when connection is made along 1p, resistance R_{np} is added to R_x and R_{mp} is added to R_3.
• The balancing condition demands that if the connection is along 1p, the ratio of resistance between n and p and between m and p, namely, R_{np} and R_{mp} respectively, will be given by

$$\frac{R_{np}}{R_{mp}} = \frac{R_1}{R_2} \qquad (1)$$

$$\frac{R_{np} + R_{mp}}{R_{mp}} = \frac{R_1 + R_2}{R_2}$$

Therefore, $R_{mp} = \frac{R_2 R_y}{R_1 + R_2}$ (2)

Similarly, $R_{np} = \frac{R_1 R_y}{R_1 + R_2}$ (3)

- Now, the balance condition for the bridge becomes,

$$(R_x + R_{np})R_2 = R_1(R_3 + R_{mp})$$

$$R_x + R_{np} = \frac{R_1}{R_2}(R_3 + R_{mp})$$

- Then substituting the values of R_{np} and R_{mp} from equations (2) and (3) into the above equation, we get

$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1}{R_2}\left(R_3 + \frac{R_2 R_y}{R_1 + R_2}\right)$$

$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_y}{R_1 + R_2}$$

$$R_x = \frac{R_1 R_3}{R_2} \quad (4)$$

- Equation (4) above is called the balance equation and it shows that the effect of resistance of the connecting lead from m to n has been removed by connecting the galvanometer to the intermediate position p.

AC BRIDGES AND THEIR APPLICATIONS

CONDITIONS FOR BRIDGE BALANCE

• An ac bridge consists of four bridge arms, a source of excitation and a null detector. The power source is an ac source and the null detector is usually a headphone.

• An ac bridge having impedences Z_1, Z_2, Z_3 and Z_4 in its arms is shown in the Fig. (1) below.

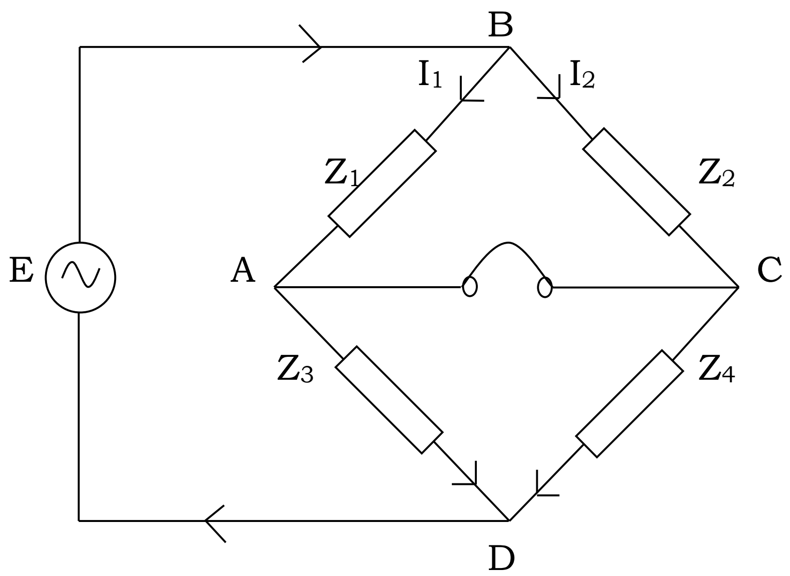

Fig. (1)

• When balance of the bridge is obtained the terminals A and C are said to be at same potential with respect to point B i.e.,

$$E_{AB} = E_{BC} \tag{1}$$

• For this condition, we can write,

$$I_1Z_1 = I_2Z_2 \tag{2}$$

Where

$$I_1 = \frac{E}{Z_1+Z_3} \tag{3}$$

and

$$I_2 = \frac{E}{Z_2+Z_4} \quad (4)$$

• Substituting the values of I_1 and I_2 from equations (3) and (4) into (2), we get

$$\frac{E}{Z_1+Z_3} Z_1 = \frac{E}{Z_2+Z_4} Z_2$$

$$\frac{Z_1}{Z_1+Z_3} = \frac{Z_2}{Z_2+Z_4}$$

therefore, $Z_1(Z_2+Z_4) = Z_2(Z_1+Z_3)$

$$Z_1Z_4 = Z_2Z_3 \quad (5)$$

• Equation (5) is called the general equation for balance of ac bridge. Similarly, we can also write this equation in terms of admittances(Y) instead of impedences as

$$Y_1Y_4 = Y_2Y_3 \quad (6)$$

• If the impedence is written in the form of $Z = \angle Z\ \theta$, then equation (5) can be written as

$$(Z_1\angle\theta_1)\ (Z_4\angle\theta_4) = (Z_2\angle\theta_2)\ (Z_3\angle\theta_3)$$

$$Z_1\ Z_4\angle(\theta_1+\theta_4) = Z_2\ Z_3\angle(\theta_2+\theta_3) \quad (7)$$

• Equation (7) shows two balance conditions. The first condition is that the magnitudes of the impedances satisfy the relationship shown by equation (5).

• It states that “The products of the magnitudes of impedances of the opposite arms must be equal”.

• The second condition requires that the phase angles of the impedances satisfy the relationship

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3 \quad (8)$$

• It states that “The sum of the phase angles of the opposite arms must be equal”.

MAXWELL BRIDGE

• The Maxwell bridge is used to measure an unknown inductance in terms of a known capacitance. The circuit for the Maxwell Bridge is shown in Fig. (1) below.

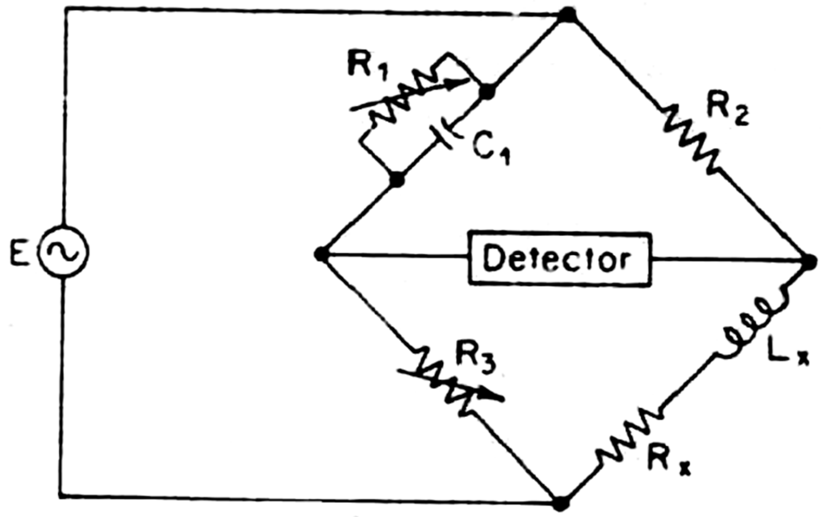

Fig. (1)

• As shown in Fig. (1), the ratio arm has parallel combination of resistance R_1 and a capacitance C_1.

• The unknown arm contains unknown inductance L_x and resistance R_x.

• The balance condition of the bridge can be given by

$$Z_1 Z_X = Z_2 Z_3$$

$$Z_X = Z_2 Z_3 Y_1 \qquad (1)$$

where $Y_1 = 1/Z_1$ = admittance

• The bridge is balanced by first adjusting R_3 for inductive balance and then by adjusting R_1 for resistive balance.

• Adjustment of R_1 disturbs the inductive balance and R_3 needs to be modified and accordingly R_1 also is modified.

• The process gives a slow convergence and the balance slowly shifts towards the actual null point. This type of balance is called the 'sliding balance'. The actual null point is obtained after few adjustments.

• From Fig. (1) we can write the impedence of the arms as

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1} + \frac{j}{X_1} = \frac{1}{R_1} + j\omega C_1 \qquad (2)$$

$$Z_2 = R_2 \ ; \quad Z_3 = R_3 \qquad (3)$$

$$Z_x = R_x + jX_L = R_x + j\omega L_x \qquad (4)$$

$$R_x + j\omega L_X = R_2 R_3 \left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x + j\omega L_X = \left(\frac{R_2 R_3}{R_1} + j\omega C_1 R_1 R_2\right)$$

• Equating the real parts of the above equation we get

$$R_x = \frac{R_2 R_3}{R_1}$$

• Equating the imaginary parts we get

$$L_x = R_2 R_3 C_1$$

• **Limitation of the Maxwell bridge** is that it is not suitable for the determination of the self-inductance of coils having quality factor $Q > 10$ or $Q < 1$. It is suitable for the coils having Q values in the range $1<Q<10$.

• This is because for high Q coils the phase angle is very nearly 90^o(positive). This requires a capacitor having phase angle very nearly equal to 90^o (positive).

• This situation demands that R_1 should be very large, which is impractical.

HAY BRIDGE

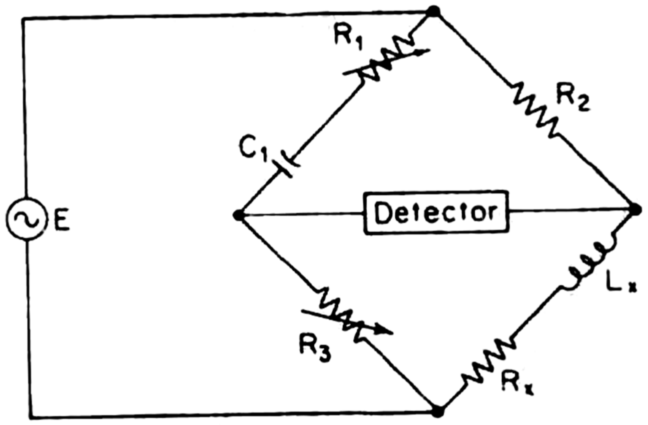

Fig. (1)

- Hay bridge is usually used to determine the unknown inductance of coils having higher Q values(>10).
- It consists of resistor R_1 in series with standard capacitor C_1 as shown in Fig. (1).
- The balance condition for the bridge is given by

$$Z_1 Z_x = Z_2 Z_3 \qquad (1)$$

- The impedances of the arms of the bridge can be given by

$$Z_1 = R_1 - jX_{C1} = R_1 - \frac{j}{\omega C_1}\ ; \quad Z_2 = R_2\ ; \quad Z_3 = R_3\ ; \qquad (2)$$

$$Z_x = R_x + jX_{Lx} = R_x + j\omega L_x \qquad (3)$$

- Substituting the values from equations (2) and (3) into equation (1), we get

$$\left(R_1 - \frac{j}{\omega C_1}\right)(R_x + j\omega L_x) = R_2 R_3$$

It can be expanded as

$$R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega L_x R_1 = R_2 R_3 \qquad (4)$$

• Separating the real and imaginary terms on both the sides of equation (4) we get

$$R_1R_x + \frac{L_x}{C_1} = R_2R_3 \qquad (5)$$

$$-\frac{jR_x}{\omega C_1} + j\omega L_xR_1 = 0$$

$$\frac{jR_x}{\omega C_1} = j\omega L_xR_1 \quad \text{or} \quad \frac{R_x}{\omega C_1} = \omega L_xR_1 \qquad (6)$$

• Here both equations (5) and (6) contain L_x and R_x and to find their values we must solve them. The solution gives the following values of L_x and R_x.

$$R_x = \frac{\omega^2C_1^2R_1R_2R_3}{1+\omega^2C_1^2R_1^2} \qquad (7)$$

And $$L_x = \frac{R_2R_3C_1}{1+\omega^2C_1^2R_1^2} \qquad (8)$$

• Now the impedance triangles for the inductive and capacitive arms can be represented as in Fig. (2).

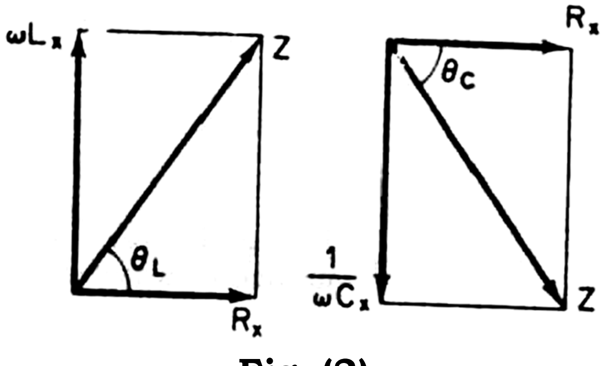

Fig. (2)

• In Fig. (2) θ_L and θ_C and the inductive and capacitive phase angles respectively. Then from the Fig. we can write

$$\tan\theta_L = \frac{X_L}{R} = \frac{\omega L_x}{R_x} = Q$$

$$\text{and } \tan\theta_C = \frac{X_C}{R} = \frac{1}{\omega C_1 R_1}$$

• When the two phase angles are equal, we have

$$\tan\theta_L = \tan\theta_C$$

$$Q = \frac{1}{\omega C_1 R_1} \qquad (9)$$

• Then L_x can be given by the equation

$$L_x = \frac{R_2 R_3 C_1}{1+\left(\frac{1}{Q}\right)^2}$$

•So for Q > 10, (1/100) <<< 1 and hence we have

$L_x = R_2R_3C_1$

• Thus for Q>10 Hay bridge is the most suitable bridge, however, for Q<10, (1/Q)2 is not negligible and hence Maxwell bridge is more suitable.

SCHERRING BRIDGE

- A Scherring Bridge is generally used to determine the unknown capacitance of a capacitor. However, it is sometimes also used to measure the insulating or dielectric properties of materials.
- The circuit arrangements for the bridge are given below in Fig. (1).

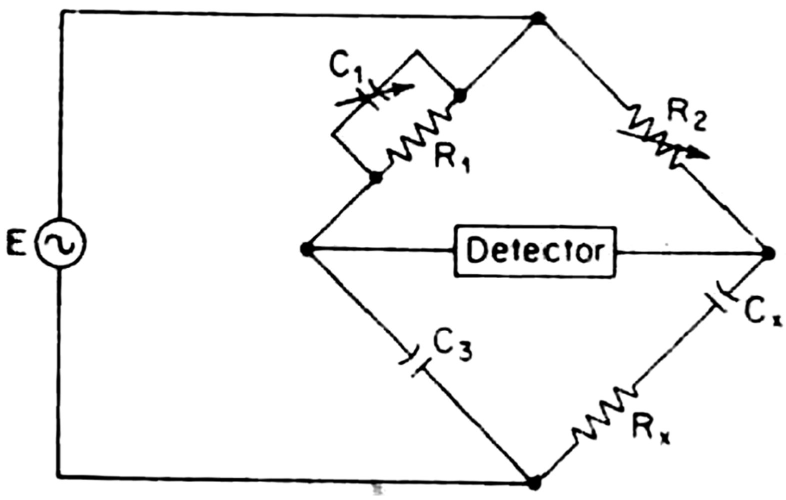

Fig. (1)

- Here, the ratio arm has the capacitor C_1 in parallel with resistor R_1.
- The balance condition for the bridge is given by the equation

$$Z_x = Z_2 Z_3 Y_1 \qquad (1)$$

- The impedances of the arms are given by

$$\frac{1}{Z_1} = Y_1 = \frac{1}{R_1} + \frac{j}{X_{C1}} = \frac{1}{R_1} + j\omega C_1 \quad ; \qquad Z_2 = R_2 \; ; \qquad (2)$$

$$Z_3 = -jX_{C3} = \frac{-j}{\omega C_3} \; ; \qquad Z_x = R_x - jX_{Cx} = R_x - \frac{j}{\omega C_X} \qquad (3)$$

- Substituting the values from equations (2) and (3) into equation (1), we get

$$R_x - \frac{j}{\omega C_X} = R_2 \left(\frac{-j}{\omega C_X}\right)\left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x - \frac{j}{\omega C_X} = \frac{R_2 C_1}{C_3} - \frac{j R_2}{\omega C_3 R_1}$$

• Then equating the real and imaginary parts in the above equation, we get

$$R_x = R_2C_1/C_3 \qquad \text{and} \qquad C_x = C_3R_1/R_2 \qquad (4)$$

• The power factor for the series RC combination is defined as the cosine of the phase angle of the circuit. It is given by

$$PF = R_x/X_x = \omega C_x R_x \qquad (5)$$

• Similarly, the dissipation factor of a series RC circuit is defined as the cotangent of the phase angle of the circuit. It is given by

$$D = \omega C_1 R_1 \qquad (6)$$

WIEN BRIDGE

• A Wien bridge is generally used to measure the frequency of the source. It is also extensively used in other applications like, harmonic distortion analyser, in audio and HF oscillators as the frequency determining element, etc.

• As shown in Fig. (1) below, the Wien bridge consists of a series RC combination in one arm and a parallel RC combination in the adjoining arm.

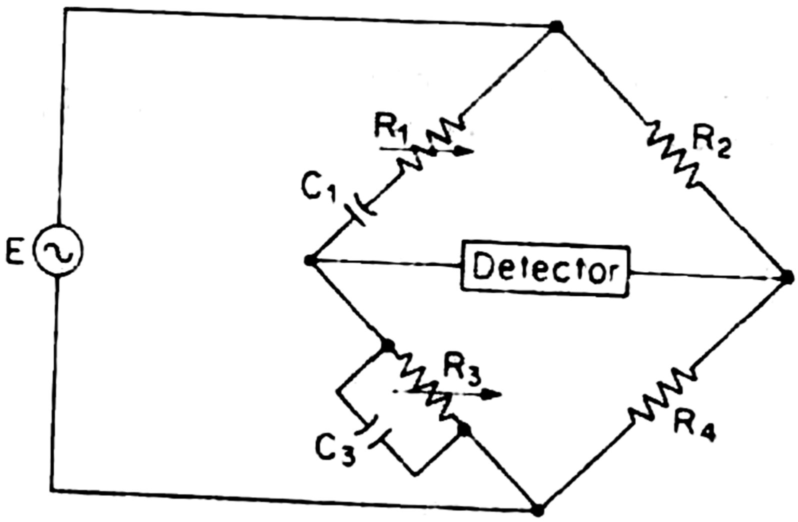

Fig. (1)

• The balance condition for the bridge is given by

$$Z_2 Z_3 = Z_1 Z_4$$

$$Z_2 = Z_1 Z_4 (1/Z_3) = Z_1 Z_4 Y_3 \qquad (1)$$

• Now the impedances of the arms are given by the following equations

$$Z_1 = R_1 - jX_{C1} = R_1 - \frac{j}{\omega C_1} \quad ; \quad Z_2 = R_2 \ ; \qquad (2)$$

$$1/Z_3 = Y_3 = \frac{1}{R_3} + \frac{j}{X_{C3}} = \frac{1}{R_3} + j\omega C_3 \quad ; \quad Z_4 = R_4 \qquad (3)$$

• Substituting the values from equations (2) and (3) into equation (1), we get

$$R_2 = \left(R_1 - \frac{j}{\omega C_1}\right) R_4 \left(\frac{1}{R_3} + j\omega C_3\right)$$

$$R_2 = \frac{R_1 R_4}{R_3} + j\omega C_3 R_1 R_4 - \frac{jR_4}{\omega C_1 R_3} + \frac{R_4 C_3}{C_1} \qquad (4)$$

• Equating the real parts we get,

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1}$$

•Dividing by R_4 on both the sides we get

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \qquad (5)$$

• Now equating the imaginary parts we get,

$$j\omega C_3 R_1 R_4 = \frac{jR_4}{\omega C_1 R_3} \qquad (6)$$

$$\omega^2 = \frac{1}{C_1 C_3 R_1 R_3} \qquad \text{where } \omega = 2\pi f$$

$$f = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}} \qquad (7)$$

• If $R_1 = R_3 = R$ and $C_1 = C_3 = C$, then we get $R_2 = 2R_4$ and

$$f = \frac{1}{2\pi RC} \qquad (8)$$

• Using the above equation, we can determine the frequency of the source. Also the bridge can be calibrated directly in terms of frequency.

MULTIPLE CHOICE QUESTIONS

1. How many arms does a Wheatstone bridge possess?

   (a) one (b) two **(c) four** (d) three

2. When a Wheatstone bridge is balanced ideally the current through the galvanometer should be

   (a) 0 A (b) 1 A (c) 1.5 A (d) 2 A

3. Wheatstone bridge is used to find unknown

   (a) resistance (b) reactance (c) inductance (d) capacitance

4. In an ac bridge the null detector is usually

   (a) a galvanometer (b) an ammeter

   (c) a head phone (d) a voltmeter

5. The inductive reactance of a coil(X_L) is given by the following equation

   (a) $X_L = (1/2\pi)fL$ (b) $X_L = 2\pi/(fL)$ **(c) $X_L = 2\pi fL$** (d) $X_L = 2\pi f/L$

6. The capacitive reactance of a capacitor (X_C) is given by the following equation

   (a) $X_C = 1/fC$ **(b) $X_C = 1/(2\pi fC)$** (c) $X_C = 2\pi/fC$ (d) $X_C = 2\pi f/C$

7. Maxwell bridge is generally used to measure the unknown

   (a) resistance (b) reactance **(c) inductance** (d) capacitance

8. Maxwell bridge is suitable for the coils having Quality factor in the range

   (a) $0.1 < Q < 1$ **(b) $1 < Q < 10$** (c) $0.01 < Q < 0.1$ (d) $10 < Q < 1$

9. Hay bridge is suitable for the coils having Quality factor

   (a) $Q > 10$ (b) $Q < 10$ (c) $Q = 10$ (d) $Q < 1$

10. Scherring bridge is used to determine the unknown

(a) resistance (b) reactance (c) inductance **(d) capacitance**

11. Wien bridge is used to measure the unknown

(a) resistance (b) reactance **(c) frequency** (d) capacitance

12. When the balance condition is satisfied in an ac bridge the ratio of ______ of the adjoining arms are equal.

(a) resistances **(b) impedances** (c) reactances (d) phase angles

13. When the balance condition is satisfied in an ac bridge the sum of the ______ of the opposite arms are equal.

(a) resistances (b) impedances (c) reactances **(d) phase angles**

14. When the balance condition is satisfied in an dc bridge the ratio of the ______ of the adjoining arms are equal.

(a) resistances (b) impedances (c) reactances (d) phase angles

15. An inductor offers a ____ resistance to the high frequency current flowing through it.

(a) low (b) zero **(c) high** (d) infinite

16. A capacitor offers a ____ resistance to the high frequency current flowing through it.

(a) low (b) zero (c) high (d) infinite

SHORT QUESTIONS : (TWO MARKS)

1. Draw the circuit diagram for the Wheatstone Bridge and write its balance condition.
2. Write the balance conditions for an ac bridge.
3. Enlist the possible sources of errors in the Wheatstone bridge measurement.
4. Explain the effect of heating of resistors and the thermo-emf on the Wheatstone bridge measurements.

5. Explain the effect of leads and contacts and the thermo-emf on the Wheatstone bridge measurements.
6. Write a note on limitations of Wheatstone bridge.
7. Discuss the limitations of Maxwell bridge.

LONG QUESTIONS

1. Discuss the basic operation of a Wheatstone bridge and derive its balance condition. **(4)**
2. Discuss in detail the various sources of errors associated with the Wheatstone bridge measurements. **(4)**
3. Obtain the Thevenin equivalent circuit of a Wheatstone bridge and hence derive the formula for the current flowing through the galvanometer. **(6)**
4. With the help of necessary circuit diagram and equations explain how Kelvin bridge is used to account for the errors introduced due to the leads and contacts. **(4)**
5. Derive the balance condition for an ac bridge with the help of the necessary circuit diagram. **(4)**
6. Describe the construction and working of Maxwell bridge and discuss the procedure to determine the unknown inductance of a coil using it. Also explain its limitations. **(7)**
7. Discuss the Hay bridge circuit in detail and explain how it overcomes the drawbacks of Maxwell bridge circuit in determining the unknown inductances. **(7)**
8. Explain the construction and working of a Scherring bridge in detail. **(5)**
9. Explain the construction and working of a Wien bridge in detail. **(5)**